After 50 years and 1 day of exponential growth…

Moore’s original paper was published in 19 April 1965

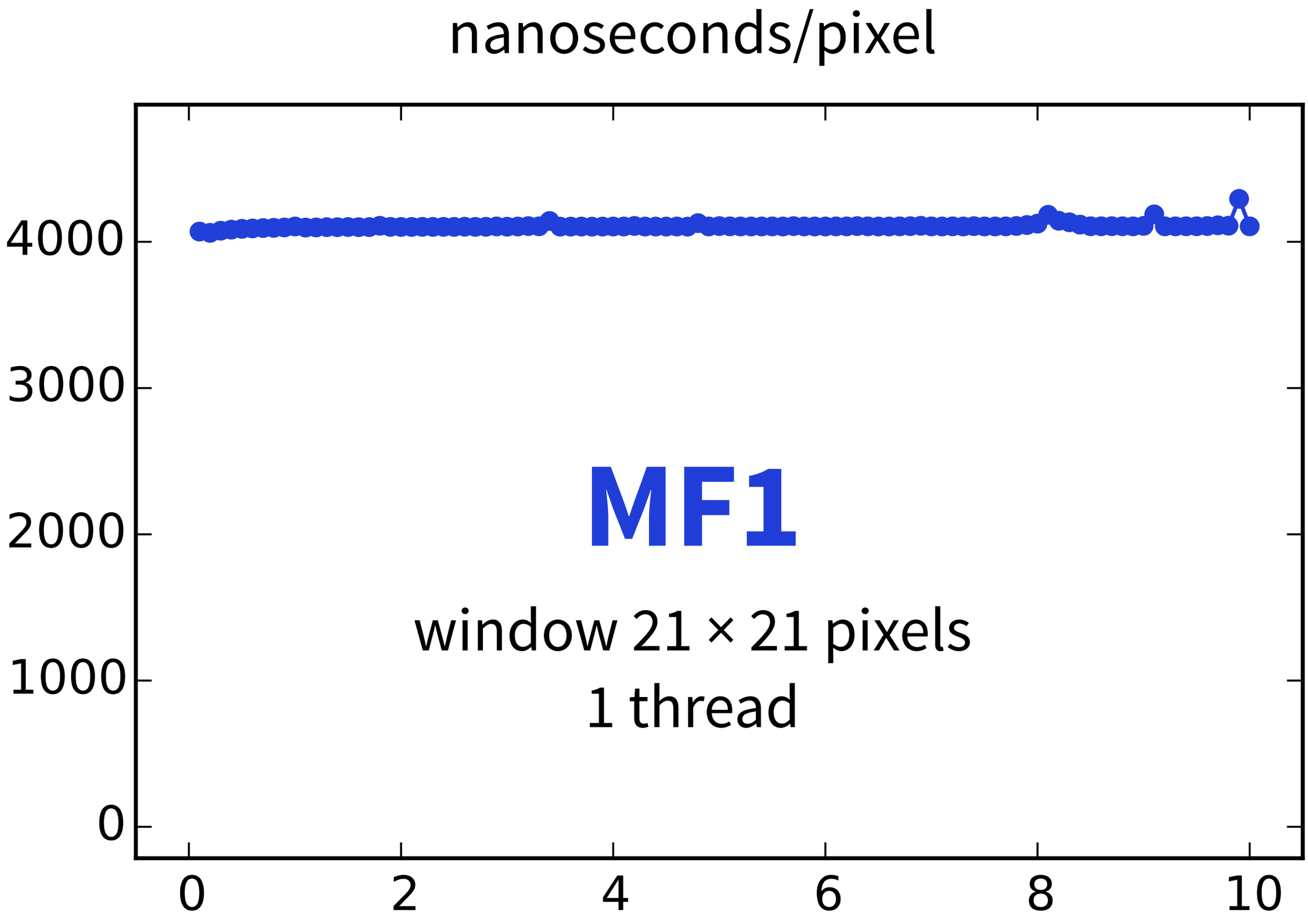
Today

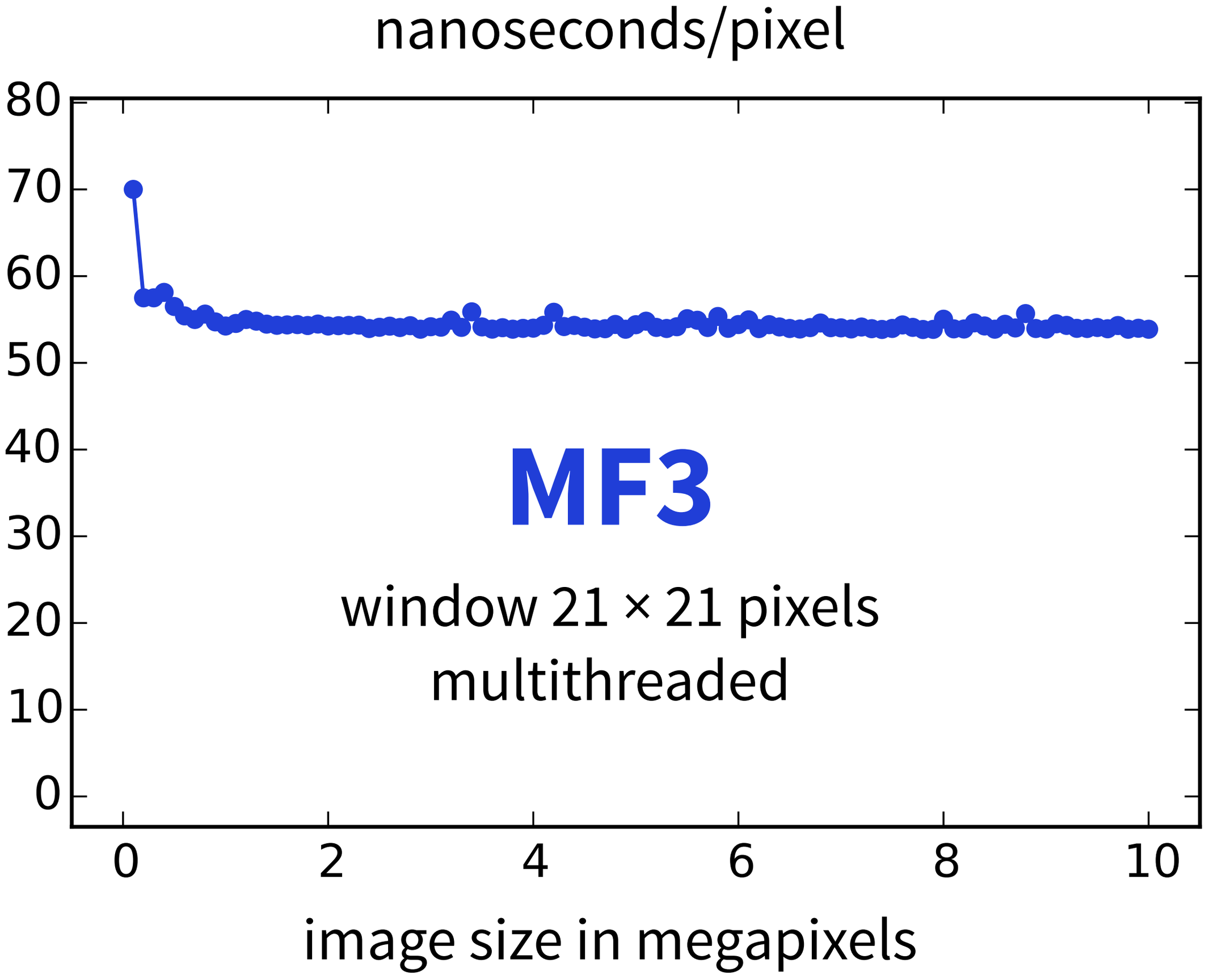
- **More about OpenMP**
- **Vector instructions (SIMD)**
- **A little bit about instruction-level parallelism and memory hierarchies (caches)**

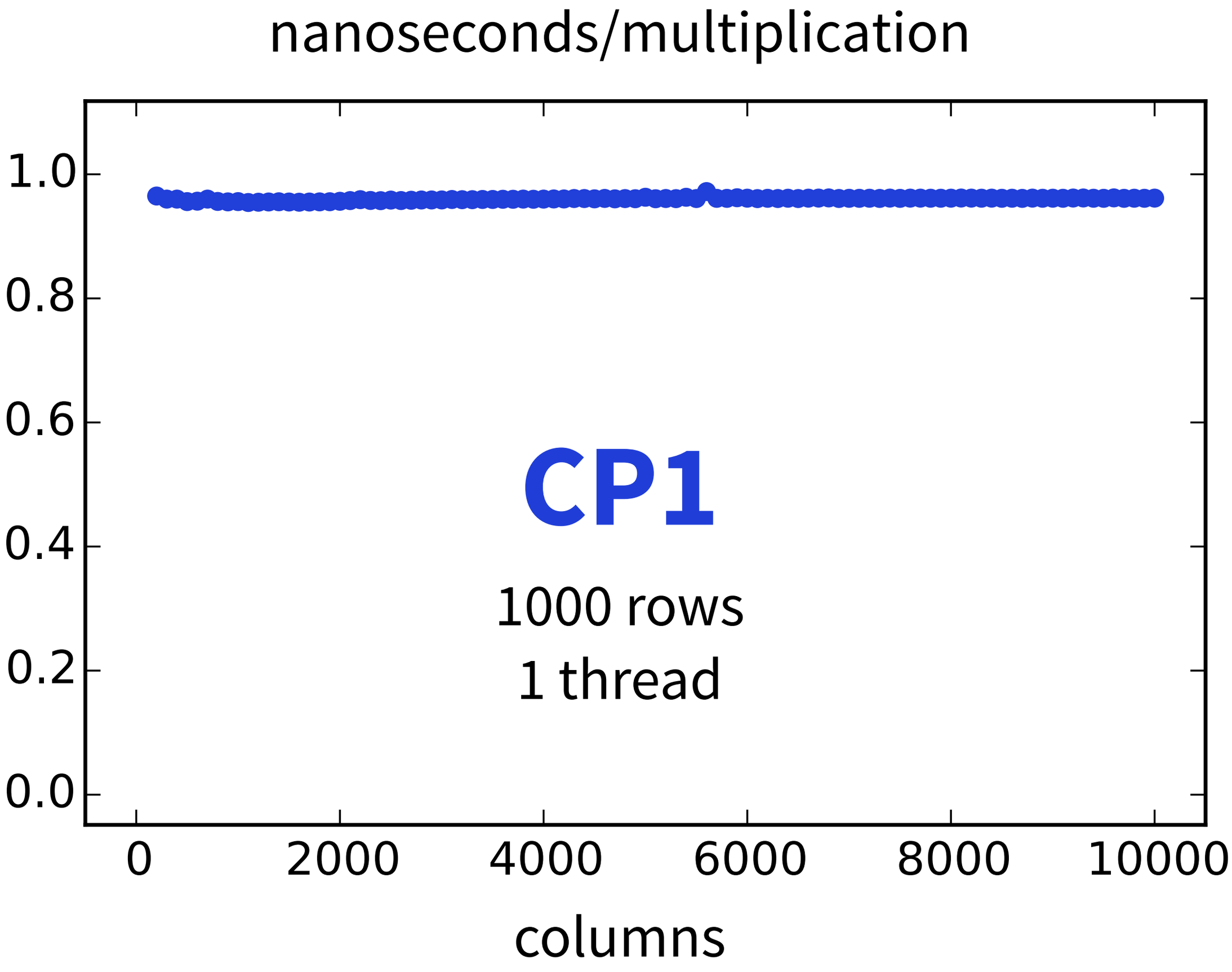
About benchmarking and reports

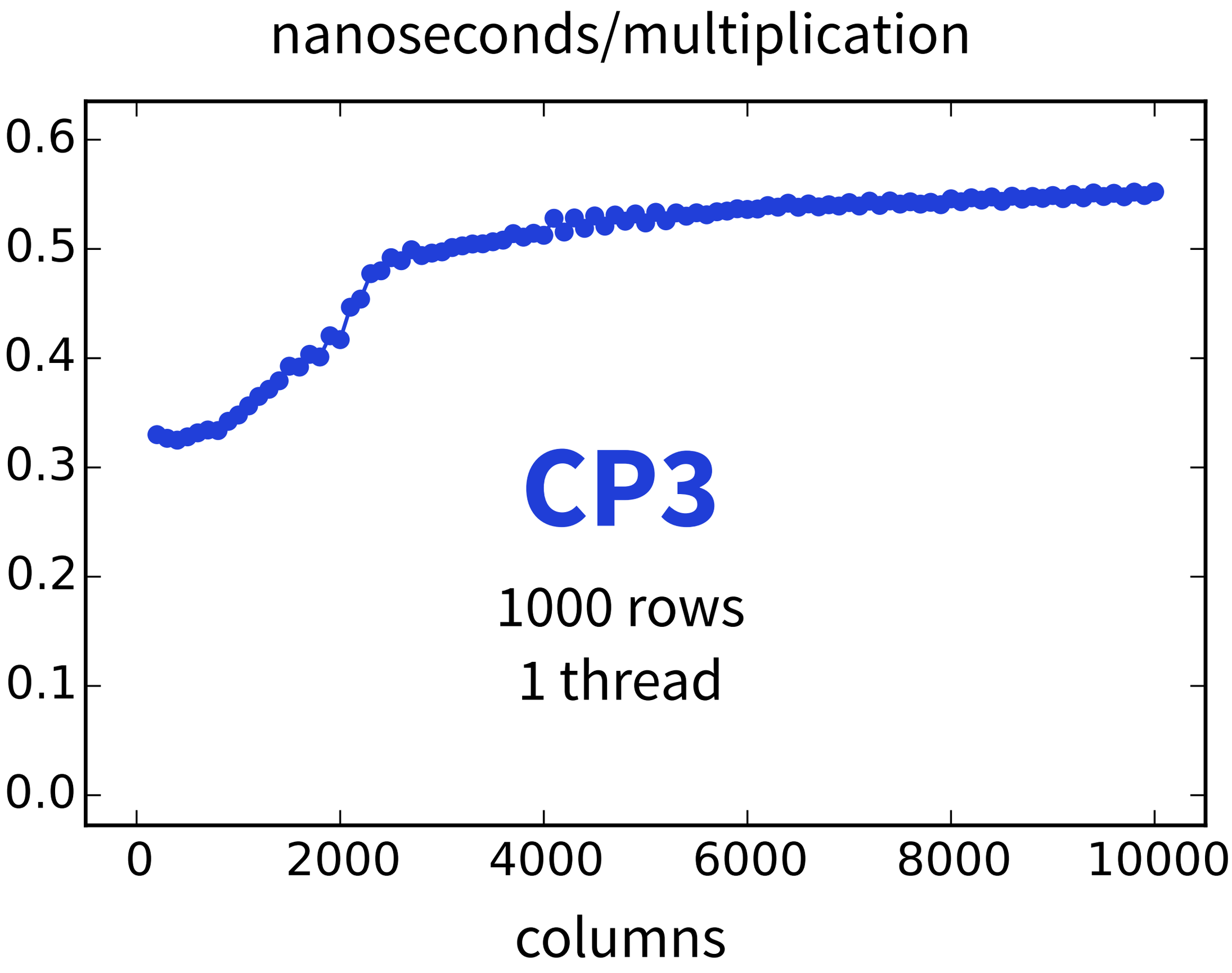
Suggestions

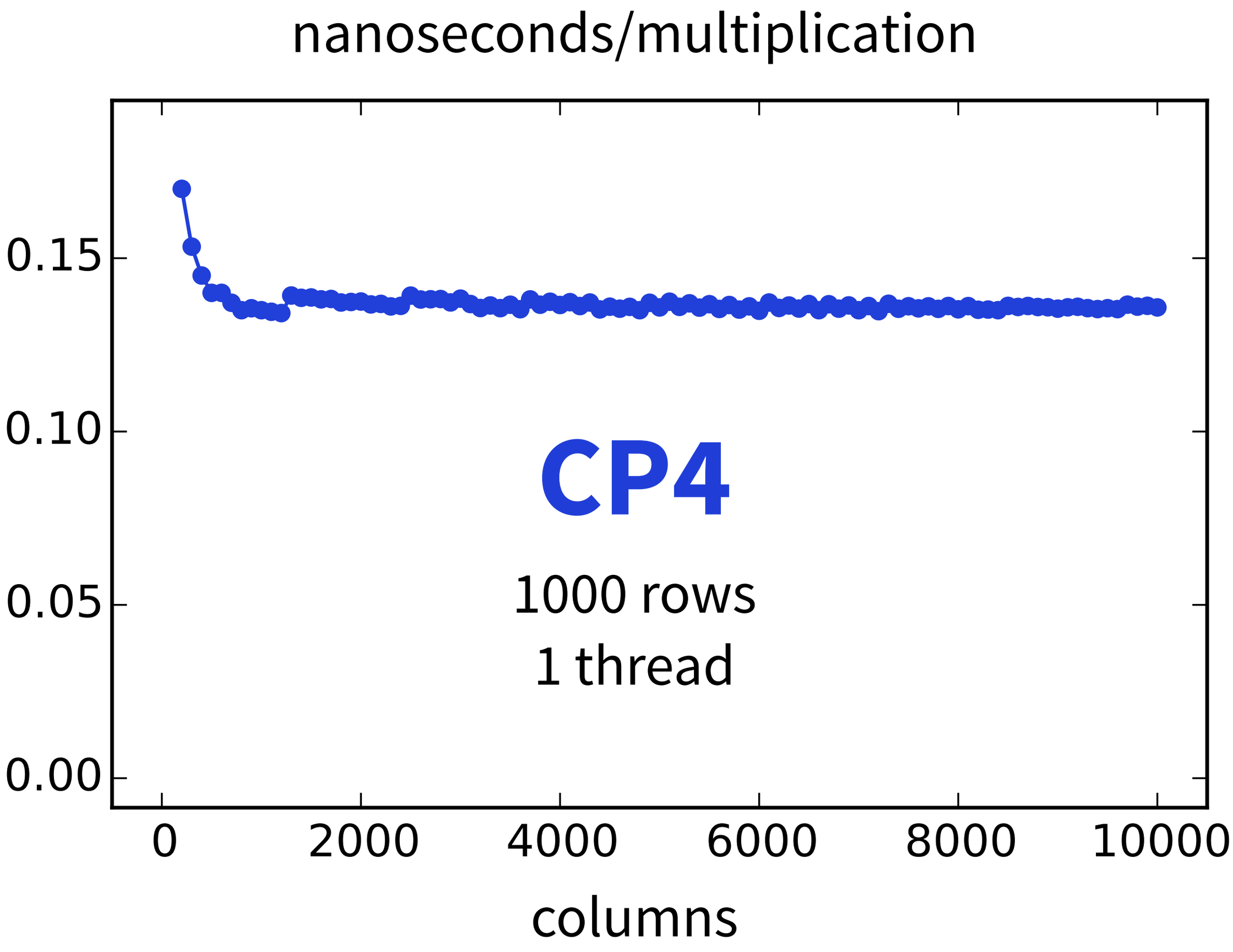
- **Calculate nanoseconds per pixel**
  - or clock cycles per pixel
  - typically nanosecond ≈ 2–3 clock cycles
- **Calculate speed (= 1/time) as a function of the number of threads**
  - compare with linear growth

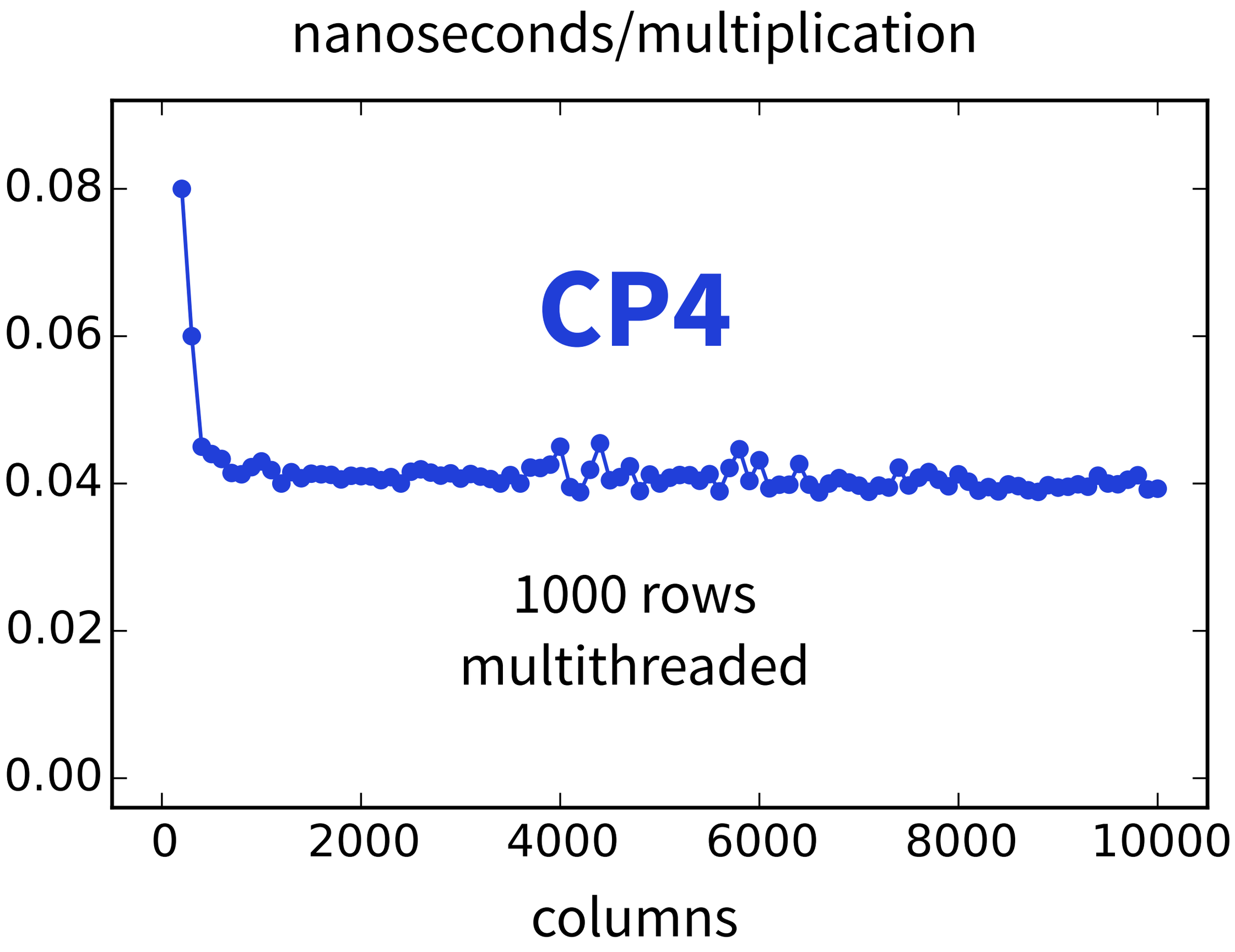

nanoseconds/pixel
4000
3000
2000
1000
0
MF1
window 21 × 21 pixels
1 thread
0
2
4
6
8
10
image size in megapixels

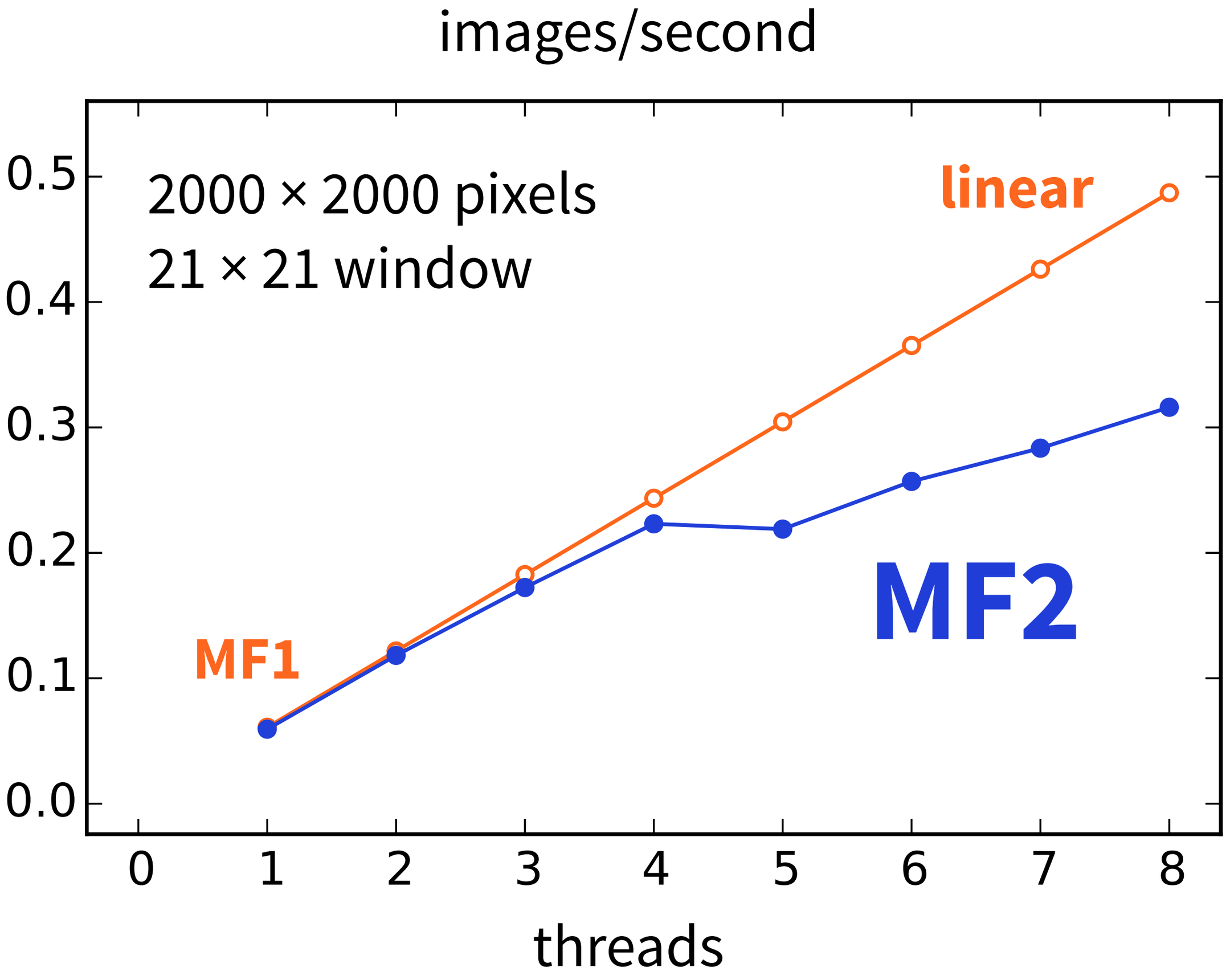

nanoseconds/pixel
80
70
60
50
40
30
20
10
0
MF3
window 21 × 21 pixels
multithreaded
0
2
4
6
8
10
image size in megapixels

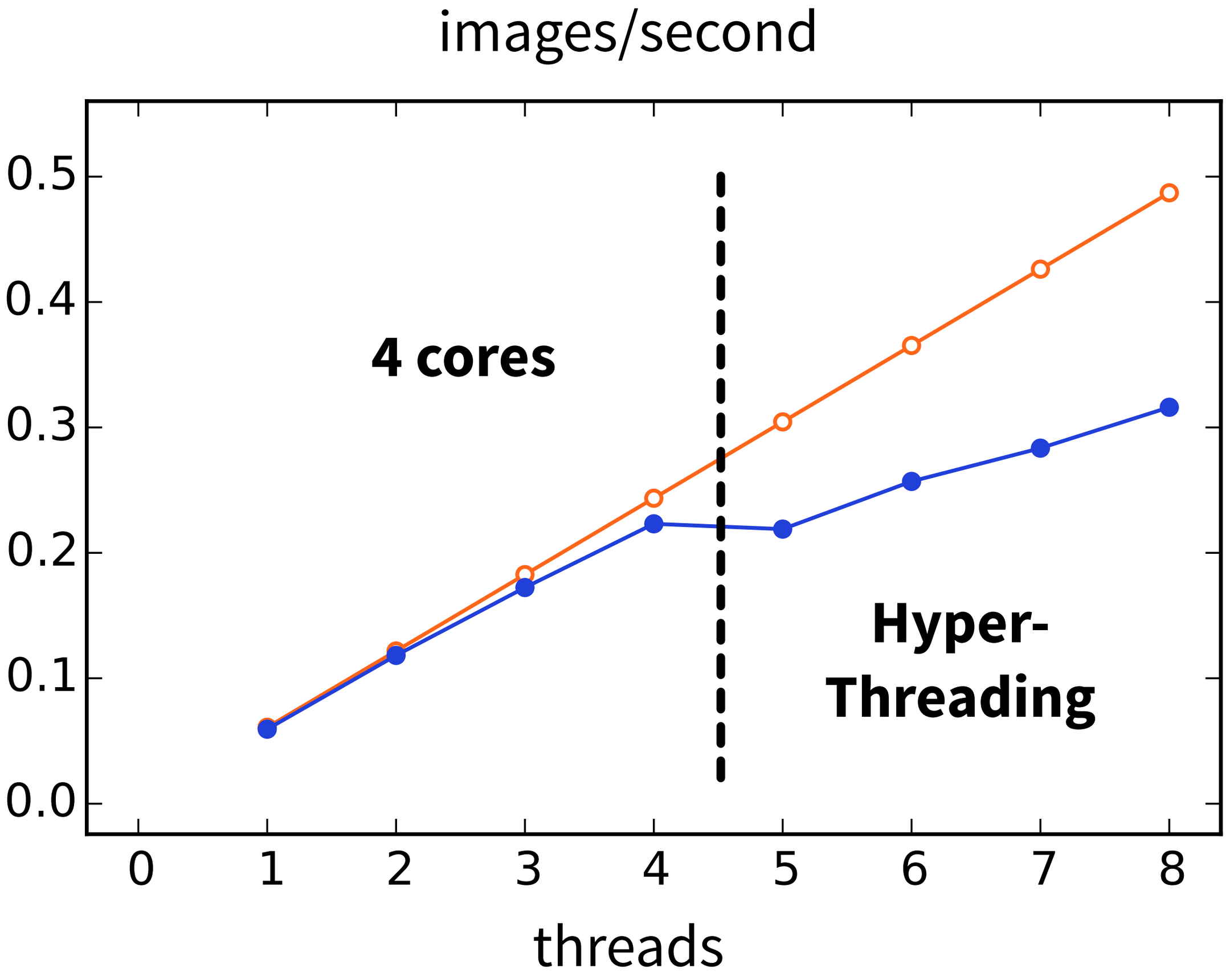

nanoseconds/multiplication
1.0
0.8
0.6
0.4
0.2
0.0
CP1
1000 rows
1 thread
0
2000
4000
6000
8000
10000
columns

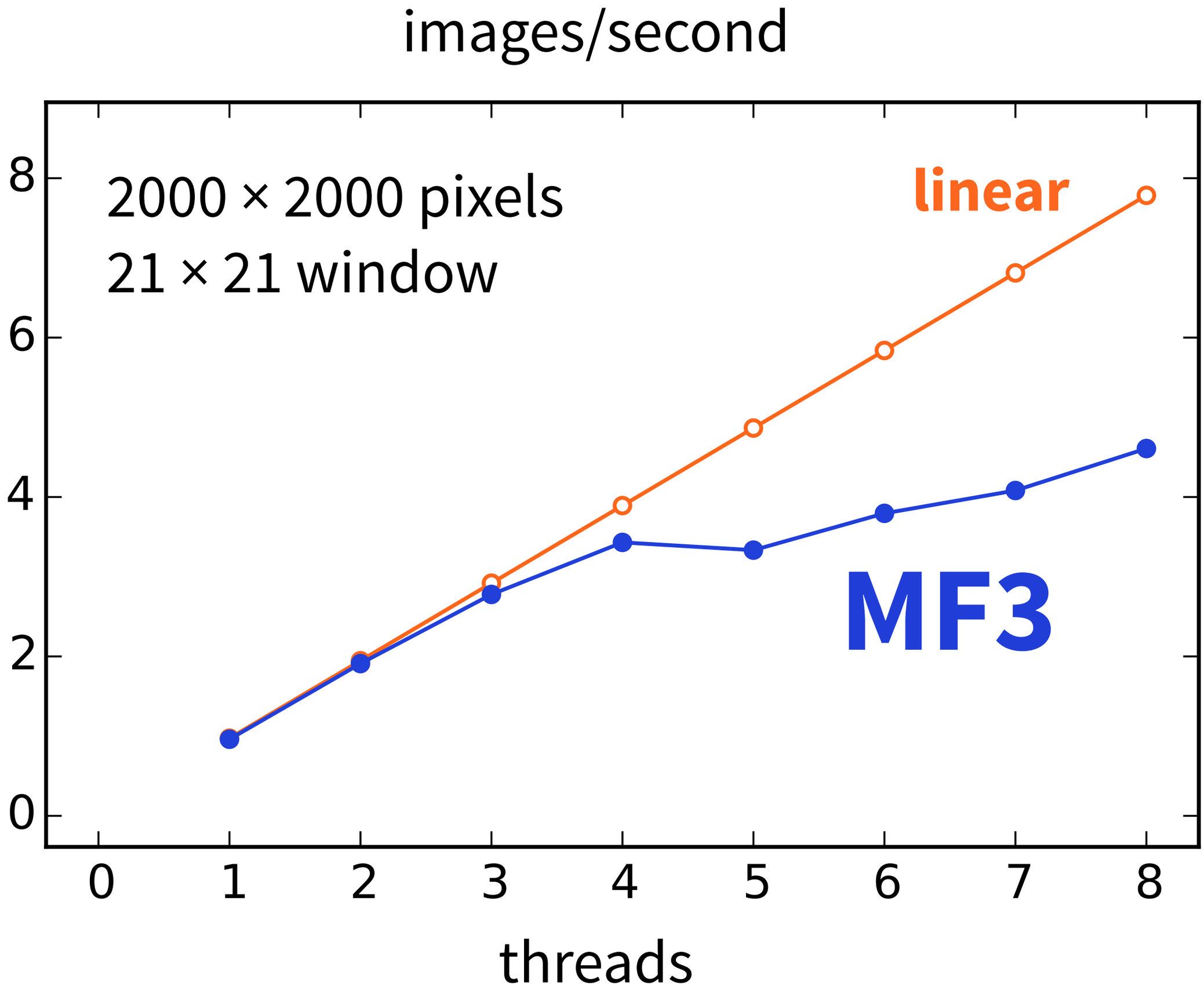

nanoseconds/multiplication
0.6
0.5
0.4
0.3
0.2
0.1
0.0
CP3
1000 rows
1 thread
0
2000
4000
6000
8000
10000
columns

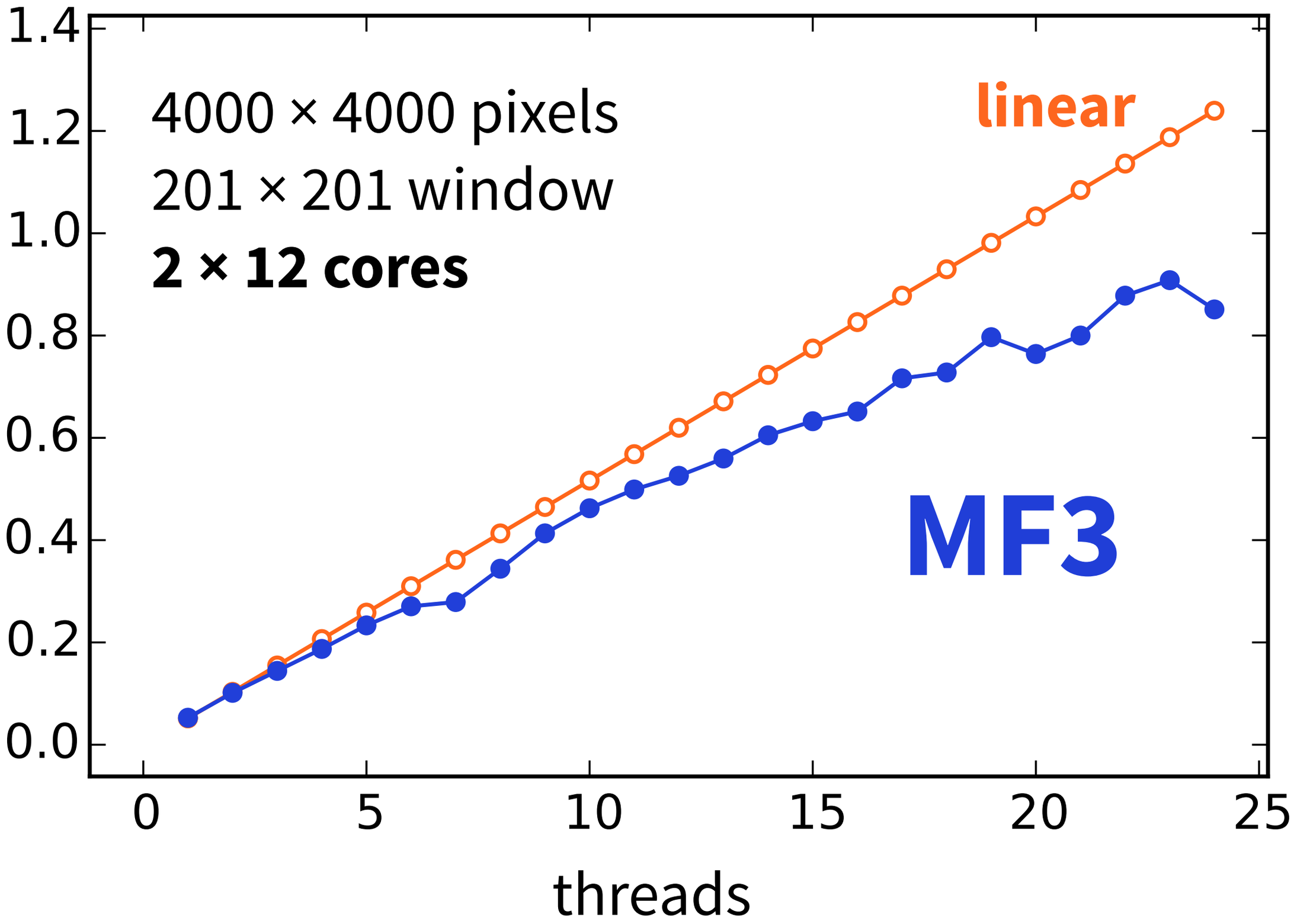

nanoseconds/multiplication
0.15
0.10
0.05
0.00
CP4
1000 rows
1 thread
0
2000
4000
6000
8000
10000
columns


nanoseconds/multiplication
0.08
0.06
0.04
0.02
0.00
CP4
1000 rows
multithreaded
0
2000
4000
6000
8000
10000
columns


images/second
2000 × 2000 pixels
21 × 21 window
linear
MF1
MF2
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
threads


images/second
0.5
0.4
0.3
0.2
0.1
0.0
4 cores
Hyper-
Threading
0
1
2
3
4
5
6
7
8
threads


images/second
2000 × 2000 pixels
21 × 21 window
linear
MF3
0
2
4
6
8
0
1
2
3
4
5
6
7
8
threads


images/second
4000 × 4000 pixels
201 × 201 window
2 × 12 cores
linear
MF3
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
threads

Calculating speedups

- **Fair baseline: a single-threaded version (compile *without* OpenMP)**
  - if you compile with OpenMP and run with `OMP_NUM_THREADS=1`, it can be much slower than a good single-threaded implementation

OpenMP: memory model

quick recap…

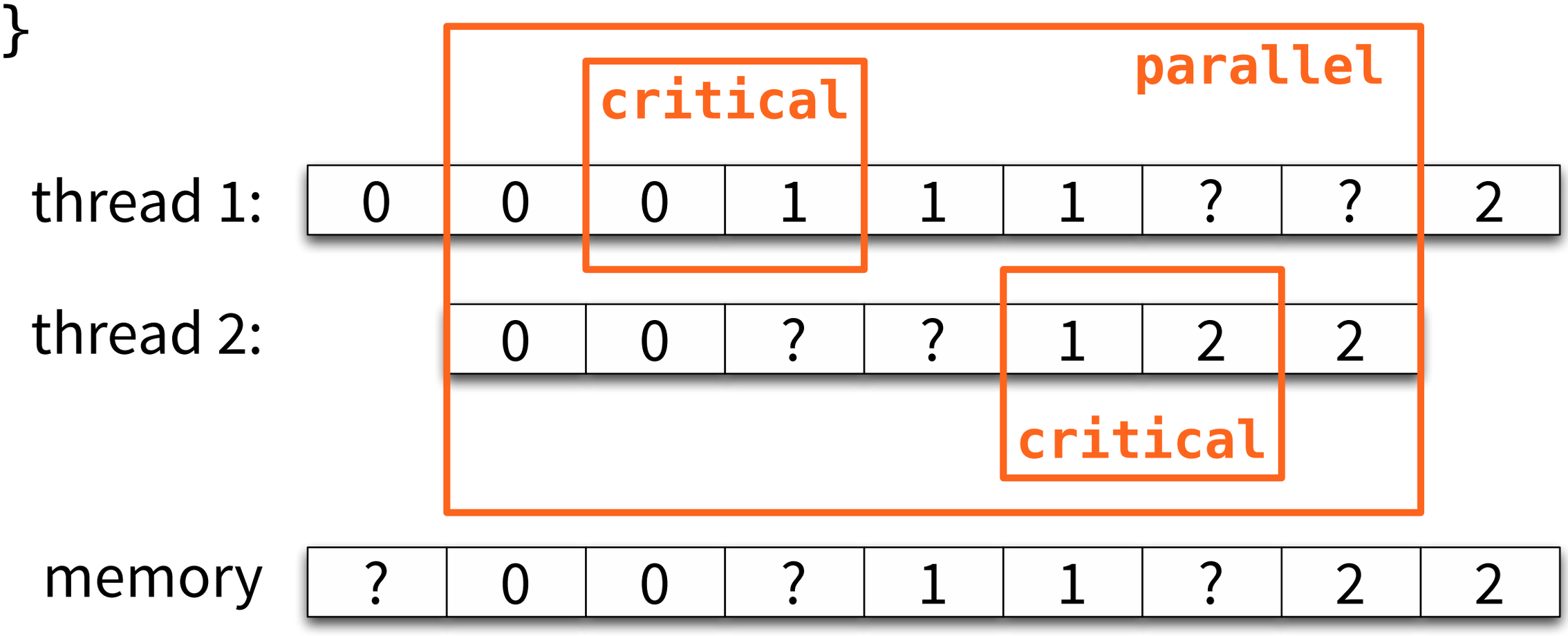
OpenMP memory model

- **Contract between programmer & system**
- **Local “*temporary view*”, global “*memory*”**
  - threads read & write temporary view
  - may or may not be consistent with memory
- **Consistency guaranteed only after a “*flush*”**

OpenMP memory model

- **Implicit “flush” e.g.:**
  - when entering/leaving “`parallel`” regions
  - when entering/leaving “`critical`” regions
- **Mutual exclusion:**
  - for “`critical`” regions

```
int a = 0;
#pragma omp parallel
{
    #pragma omp critical
     {
          a += 1;
     }
}
```



Simple rules

- **Permitted (without explicit synchronisation):**
  - *multiple threads reading*, no thread writing
  - *one thread writing*, same thread reading
- **Forbidden (without explicit synchronisation):**
  - *multiple threads writing*
  - *one thread writing, another thread reading*

Simple rules

- **Smallest meaningful unit = array element**
- **Many threads can access the same array**
- **Just be careful if they access the same array element**
  - even if you try to manipulate different bits

OpenMP: variables

private or shared?

Two kinds of variables

- **Shared variables**
  - shared among all threads
  - be very careful with data races!
- **Private variables**
  - each thread has its own variable
  - safe and easy

```
// shared variable
int sum_shared = 0;
#pragma omp parallel
{
    // private variables (one for each thread)
    int sum_local = 0;
    #pragma omp for nowait
    for (int i = 0; i < 10; ++i) {
        sum_local += i;
    }
    #pragma omp critical
    {
        sum_shared += sum_local;
    }
}
print(sum_shared);
```

```
// OK:
for (int i = 0; i < n; ++i) {
    float tmp = x[i];
    y[i] = tmp * tmp;
}

// OK:
float tmp;
for (int i = 0; i < n; ++i) {
    tmp = x[i];
    y[i] = tmp * tmp;
}
```

```
// OK:
#pragma omp parallel for
for (int i = 0; i < n; ++i) {
    float tmp = x[i];
    y[i] = tmp * tmp;
}
```

```
// Bad (data race):
float tmp;
#pragma omp parallel for
for (int i = 0; i < n; ++i) {
    tmp = x[i];
    y[i] = tmp * tmp;
}
```

```
// OK (just unnecessarily complicated):
#pragma omp parallel
{
    float tmp;
    #pragma omp for
    for (int i = 0; i < n; ++i) {
        tmp = x[i];
        y[i] = tmp * tmp;
    }
}
```

Two kinds of variables

- **Shared variables and private variables**
- **If necessary, you can customise this:**
  - `#pragma omp parallel private(x)`
  - `#pragma omp parallel shared(x)`
  - `#pragma omp parallel firstprivate(x)`
- **Seldom needed, defaults usually fine**

Best practices

- **Use subroutines!**
  - much easier to avoid accidents with shared variables this way
- **Keep the function with #pragmas as short as possible**
  - just e.g. call another function in a for loop

OpenMP: critical sections

… and atomics

```
//  Good, no critical section needed:
#pragma omp parallel for
for (int i = 0; i < 10000000; ++i) {
    ++v[i];
}


//  Bad, very slow:
#pragma omp parallel for
for (int i = 0; i < 10000000; ++i) {
    #pragma omp critical
    {
        ++v[i];
    }
}
```

```
//  Good, no critical section needed:
#pragma omp parallel for
for (int i = 0; i < 10000000; ++i) {
    ++v[i];
}
```

4 ms

```
//  Bad, very slow:
#pragma omp parallel for
for (int i = 0; i < 10000000; ++i) {
    #pragma omp critical
    {
        ++v[i];
    }
}
```

40 000 ms

```
//  Bad — no data race but undefined output:
int a = 0;
#pragma omp parallel
{
    int b;
    #pragma omp critical
    {
        b = a;
    }
    ++b;
    #pragma omp critical
    {
        a = b;
    }
}
```

```
// OK:
int a = 0;
#pragma omp parallel
{
    int b;
    #pragma omp critical
    {
        b = a;
        ++b;
        a = b;
    }
}
```

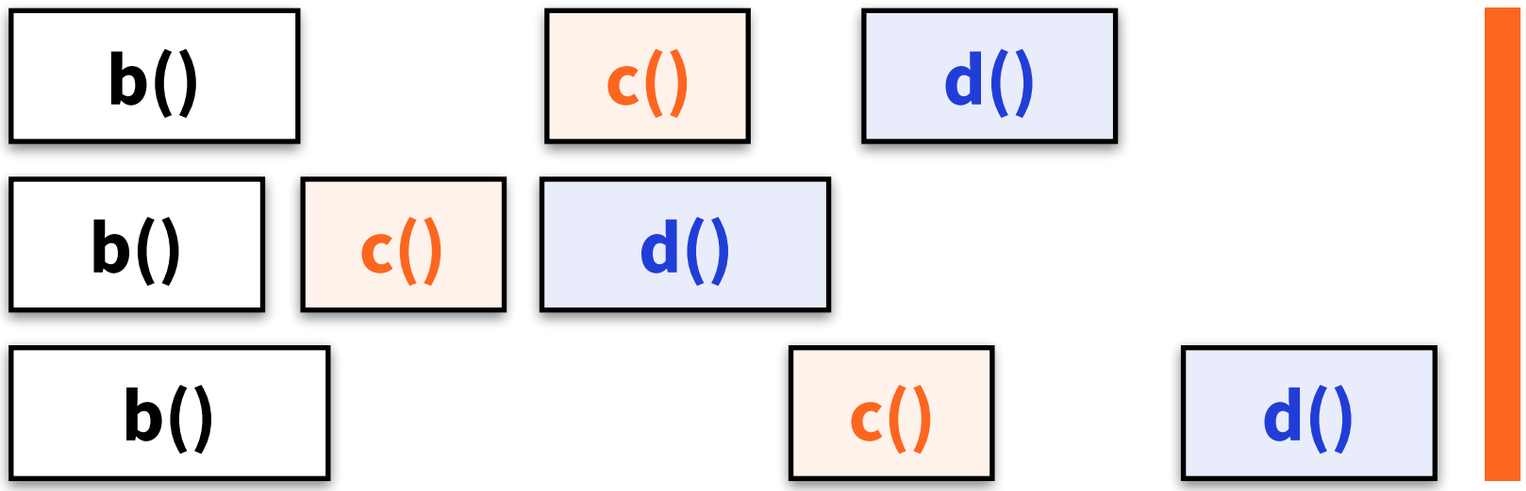
```
//  Bad: same output file
#pragma omp parallel for
for (int i = 0; i < 10; ++i) {
    int v = calculate(i);
    std::cout << v << std::endl;
}

//  OK (but no guarantees on the order of lines)
#pragma omp parallel for
for (int i = 0; i < 10; ++i) {
    int v = calculate(i);
    #pragma omp critical
    {
        std::cout << v << std::endl;
    }
}
```

Naming critical sections

- **You can give names to critical sections:**
  - `#pragma omp critical (myname)`
- **Different threads can enter simultaneously critical sections with different names**
- **No name = the same name**

```
#pragma omp parallel for
for (int i = 0; i < 10; ++i) {
    b();
    #pragma omp critical (xxx)
    {
        c();
    }
    #pragma omp critical (yyy)
    {
        d();
    }
}
```



```
#pragma omp parallel for
for (int i = 0; i < 10; ++i) {
    int v = calculate(i);
    #pragma omp critical (result)
    {
        result += v;
    }
    #pragma omp critical (output)
    {
        std::cout << v << std::endl;
    }
}
```

Atomic operation

- **Like a tiny critical section**
- **Very restricted:**
  just for e.g. updating a single variable
- **Much more efficient**

```
for (int i = 0; i < n; ++i) {
    int l = v[i] % m;
    ++p[l];
}
```

200 ms

```
#pragma omp parallel for
for (int i = 0; i < n; ++i) {
    int l = v[i] % m;
    #pragma omp atomic
    ++p[l];
}
```

70 ms

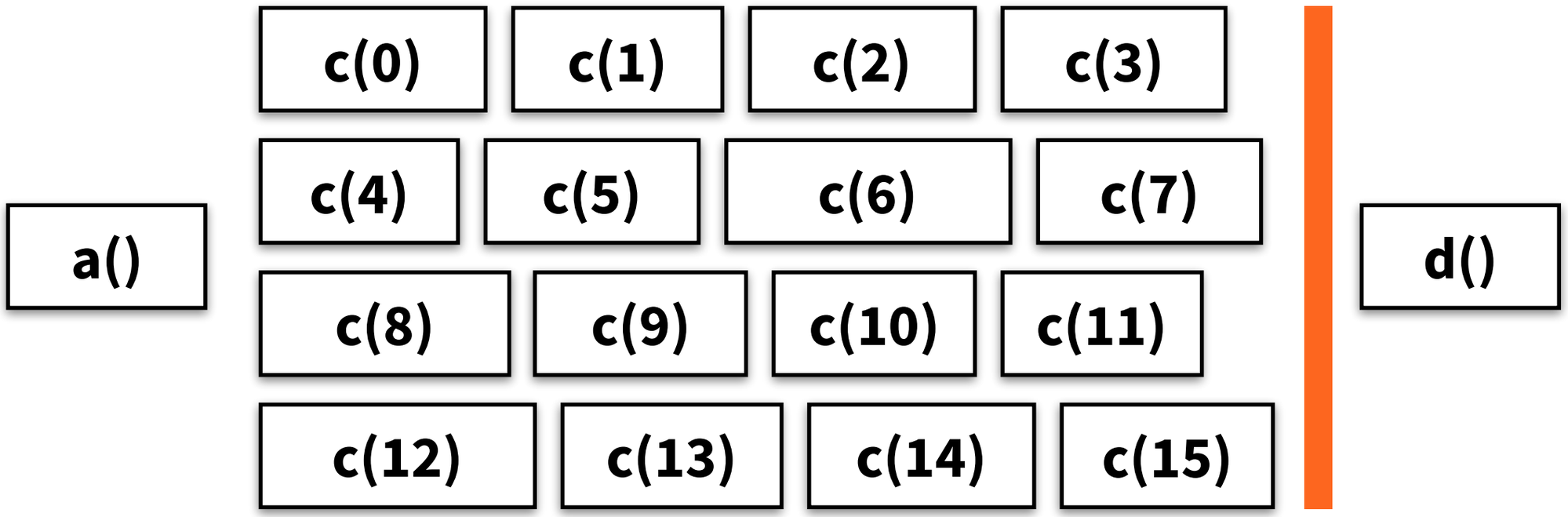
```
#pragma omp parallel for
for (int i = 0; i < n; ++i) {
    int l = v[i] % m;
    #pragma omp critical
    { ++p[l]; }
}
```

40 000 ms

OpenMP: scheduling

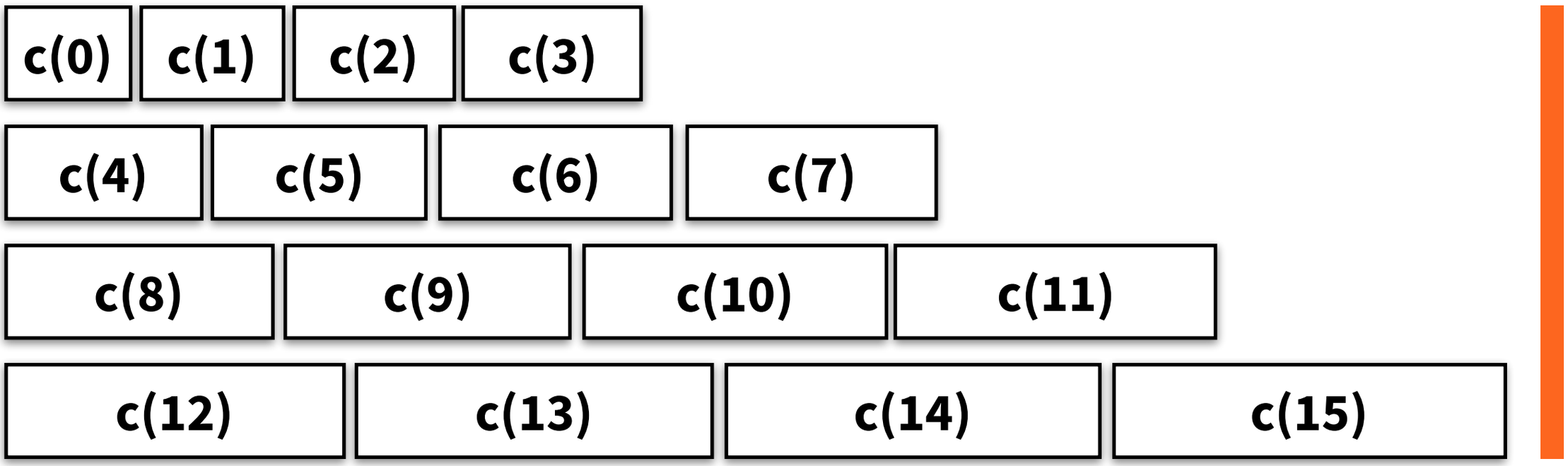
#pragma omp for

```
a();
#pragma omp parallel for
for (int i = 0; i < 16; ++i) {
    c(i);
}
d();
```

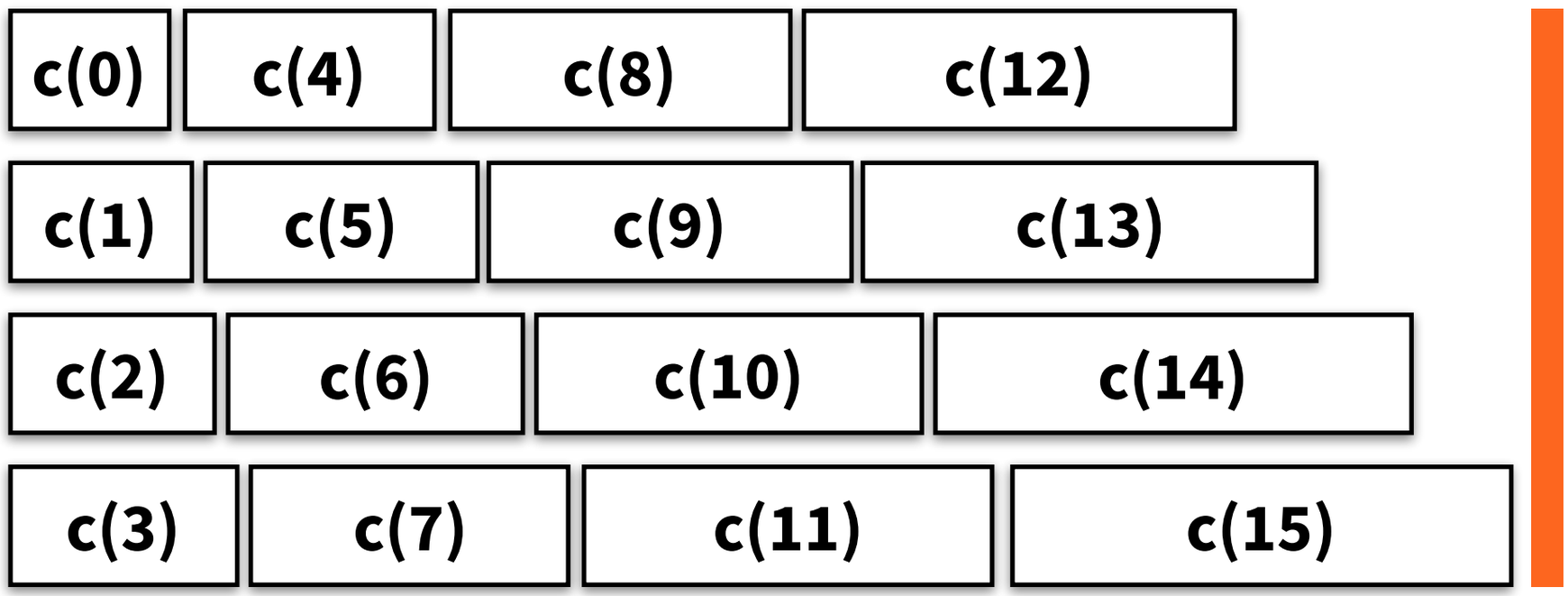


```
//  Good memory locality:
//  each thread scans a consecutive part of array
#pragma omp parallel for
for (int i = 0; i < n; ++i) {
    c(x[i]);
}
```

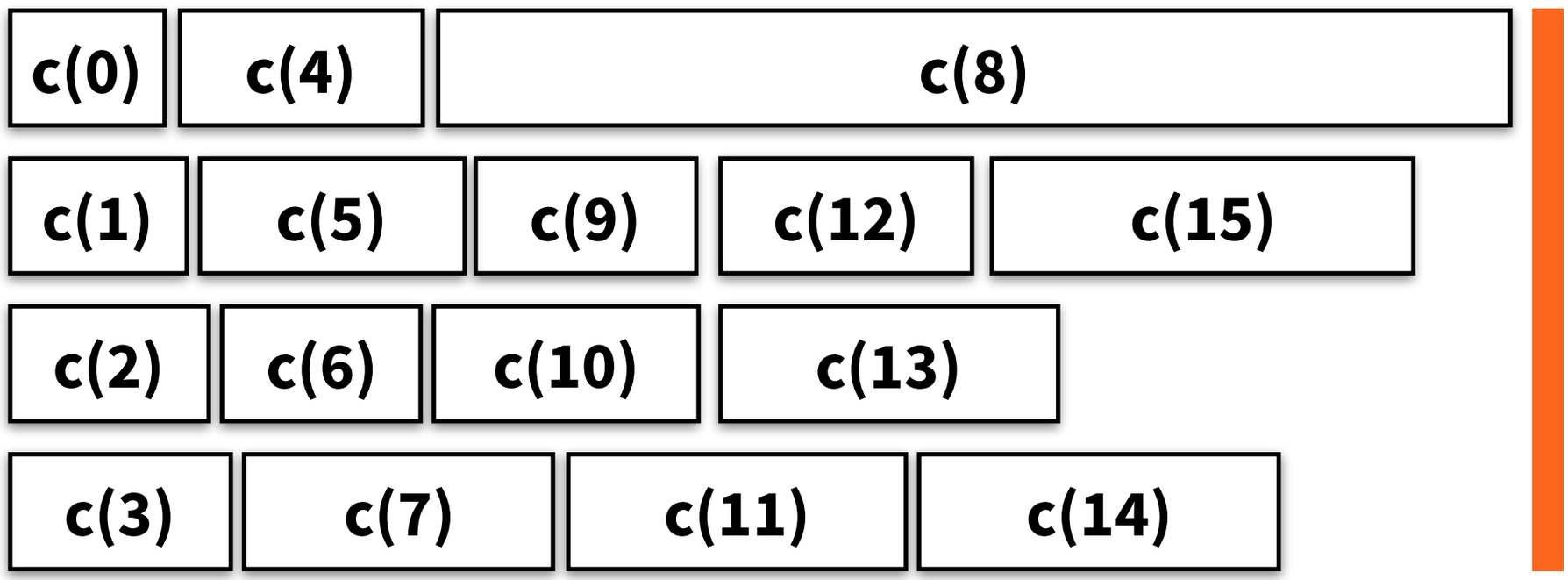
```
a();
#pragma omp parallel for
for (int i = 0; i < 16; ++i) {
    c(i);
}
d();
```



```
a();
#pragma omp parallel for schedule(static,1)
for (int i = 0; i < 16; ++i) {
    c(i);
}
d();
```



```
a();
#pragma omp parallel for dynamic
for (int i = 0; i < 16; ++i) {
    c(i);
}
d();
```



OpenMP scheduling

- **Performance, $n = 100\,000\,000$:**
  - sequential: **50 ms**
  - parallel: **50 ms**
  - `schedule(static,1)`: **200 ms**
  - `schedule(dynamic)`: **4000 ms**

```
for (int i = 0; i < n; ++i) ++v[i];
```

OpenMP scheduling

- **Performance, $n = 100\,000\,000$:**
  - sequential: **800 ms**
  - parallel: **300 ms**
  - `schedule(static,1)`: **300 ms**
  - `schedule(dynamic)`: **4000 ms**

```
for (int i = 0; i < n; ++i) v[i] = sqrt(i);
```

OpenMP: reductions

… just a convenient shorthand

```
int g = 0;
#pragma omp parallel
{
    int l = 0;
    #pragma omp for
    for (int i = 0; i < n; ++i) {
        l += v[i];
    }
    #pragma omp atomic
    g += l;
}
```

≈

```
int g = 0;
#pragma omp parallel for reduction(+:g)
for (int i = 0; i < n; ++i) {
    g += v[i];
}
```

Vector instructions

Vector instructions

- **1997: MMX**, 64-bit registers MM0 … MM7
- **1999: SSE**, 128-bit registers XMM0 … XMM7
- **2011: AVX**, 256-bit registers YMM0 … YMM15
- **soon: AVX-512**, 512-bit registers ZMM0…ZMM31

Vector instructions

- **Hundreds of machine-language instructions**
- interpret AVX registers as vectors
- “horizontal add with saturation”
- “conditional dot product”
- “sum of absolute differences”
- “fused multiply and add” …

Vector instructions

- **Hundreds of machine-language instructions**
- **Directly available via compiler intrinsics and built-in functions, if needed**
  - `c = _mm256_hadd_pd(a, b);`
  - `c = __builtin_ia32_haddpd256(a, b);`
  - see the course web site for pointers

Vector instructions

- **Hundreds of machine-language instructions**
- **Directly available via compiler intrinsics and built-in functions, if needed**
- **In many cases we do not need to worry about low-level details**

Vector types in GCC

```
typedef double double4_t __attribute__
    ((__vector_size__ (4*sizeof(double))));

//  Now these are almost equivalent:
double4_t a;
double a[4];
```

Vector types in GCC

```
typedef float float8_t __attribute__
    ((__vector_size__ (8*sizeof(float))));

//  Now these are almost equivalent:
float8_t a;
float a[8];
```

Vector types in GCC

```
// Can address individual elements as usual:
float8_t a;
for (int i = 0; i < 8; ++i) {
    a[i] = 123.0 + i;
}
```

Vector types in GCC

```
// Can address individual elements as usual:
float8_t a[3];
for (int j = 0; j < 3; ++j) {
    for (int i = 0; i < 8; ++i) {
        a[j][i] = 123.0 + i;
    }
}
```

Vector types in GCC

```
// Operations on entire vectors:
float8_t a, b, c;
a += b * c;

// Same as:
for (int i = 0; i < 8; ++i) {
    a[i] += b[i] * c[i];
}
```

Vector types in GCC

```
// Operations on entire vectors, also with scalars:
float8_t a, b, c;
a += b * c / 3 + 2;

// Same as:
for (int i = 0; i < 8; ++i) {
    a[i] += b[i] * c[i] / 3 + 2;
}
```

Vector types in GCC

- **Always available**
- **Compiler uses special vector registers and instructions whenever possible**
- **Remember to specify the architecture**
  - `g++ -march=native`

Memory alignment

- **Vector data always properly aligned**
  - memory address divisible by sizeof(vector)
- **Compiler takes care of this for local variables allocated from stack**
- **You take care of this for arrays allocated from heap**

Memory alignment

- `malloc()` **not necessarily good enough**
- `posix_memalign()` **to allocate memory,** `free()` **to release**
- **See** `common/vector.*` **for helper functions**
  - `float8_alloc()`, `double4_alloc()`

```
double4_t* x = double4_alloc(n);
double4_t* y = double4_alloc(n);

for (int i = 0; i < n; ++i) {
    for (int j = 0; j < 4; ++j) {
        x[i][j] = ...;
    }
}
```

```
for (int i = 0; i < n; ++i) {
    double4_t z = x[i];
    y[i] = z * z;
}

free(x);
free(y);
```

Memory alignment

- **CPU vector instructions:**
  require proper alignment
- **Vector types: promise of proper alignment**
- **C compiler can safely generate**
  vector instructions

```
for (int i = 0; i < n; ++i) {
    double4_t z = x[i];
    y[i] = z * z;
}
```

```
L42:
   vmovapd (%rbx,%rax), %ymm0
   vmulpd  %ymm0, %ymm0, %ymm0
   vmovapd %ymm0, (%r12,%rax)
   addq    $32, %rax
   cmpq    %rdx, %rax
   jne     L42
```

“**…pd**” = packed doubles = vector of doubles

“**ymm…**” = 256-bit register

How to exploit vector extensions

How to exploit vector instructions?

- **Needs some creativity!**
- **Design your algorithm so that you can do the *same* operation for many items, *in parallel***
- **Often some *preprocessing* & *postprocessing* needed:** convert input data to suitable vectors and back

```
//  Goal: sum of squares
s = x[0]*x[0] + ... + x[n-1]*x[n-1];

//  Preprocessing: pack to vectors
v[0] = { x[0], x[1], x[2], x[3] };
v[1] = { x[4], x[5], x[6], x[7] };
...
//  Pad last vector with zeroes if needed
v[m-1] = { x[n-2], x[n-1], 0, 0 };

//  Calculation: each component independently in parallel
y = v[0]*v[0] + ... + v[m-1]*v[m-1];

//  Postprocessing: combine components
s = y[0] + y[1] + y[2] + y[3];
```

Other examples

- **Interleave input rows:**
  in each vector, element *i* comes from row *i*
  we can then process multiple rows in parallel

- **Multidimensional input:**
  (*red*, *green*, *blue*)-triples in digital images,
  multiple channels in digital audio

Efficient use of vector instructions

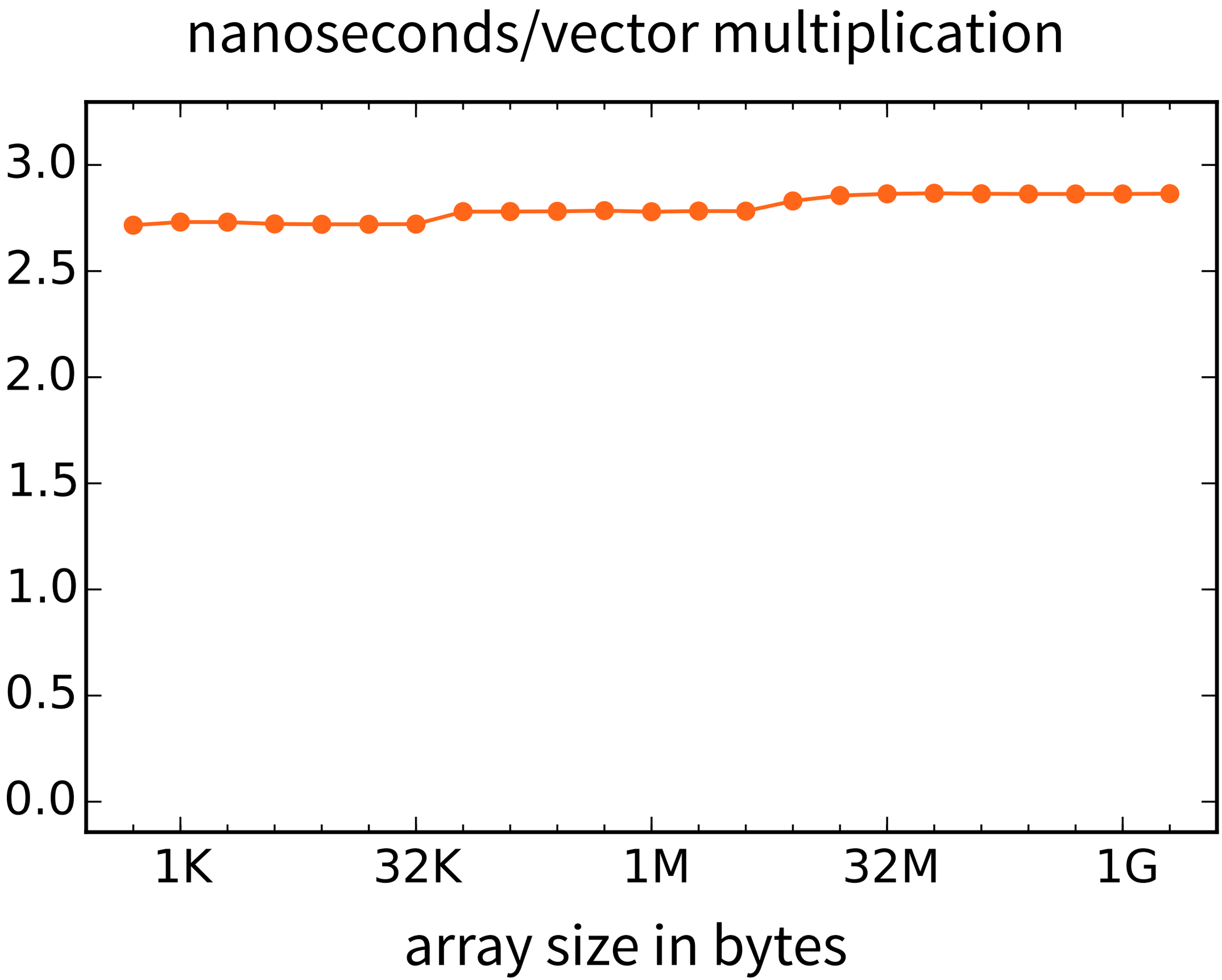
— instruction-level parallelism

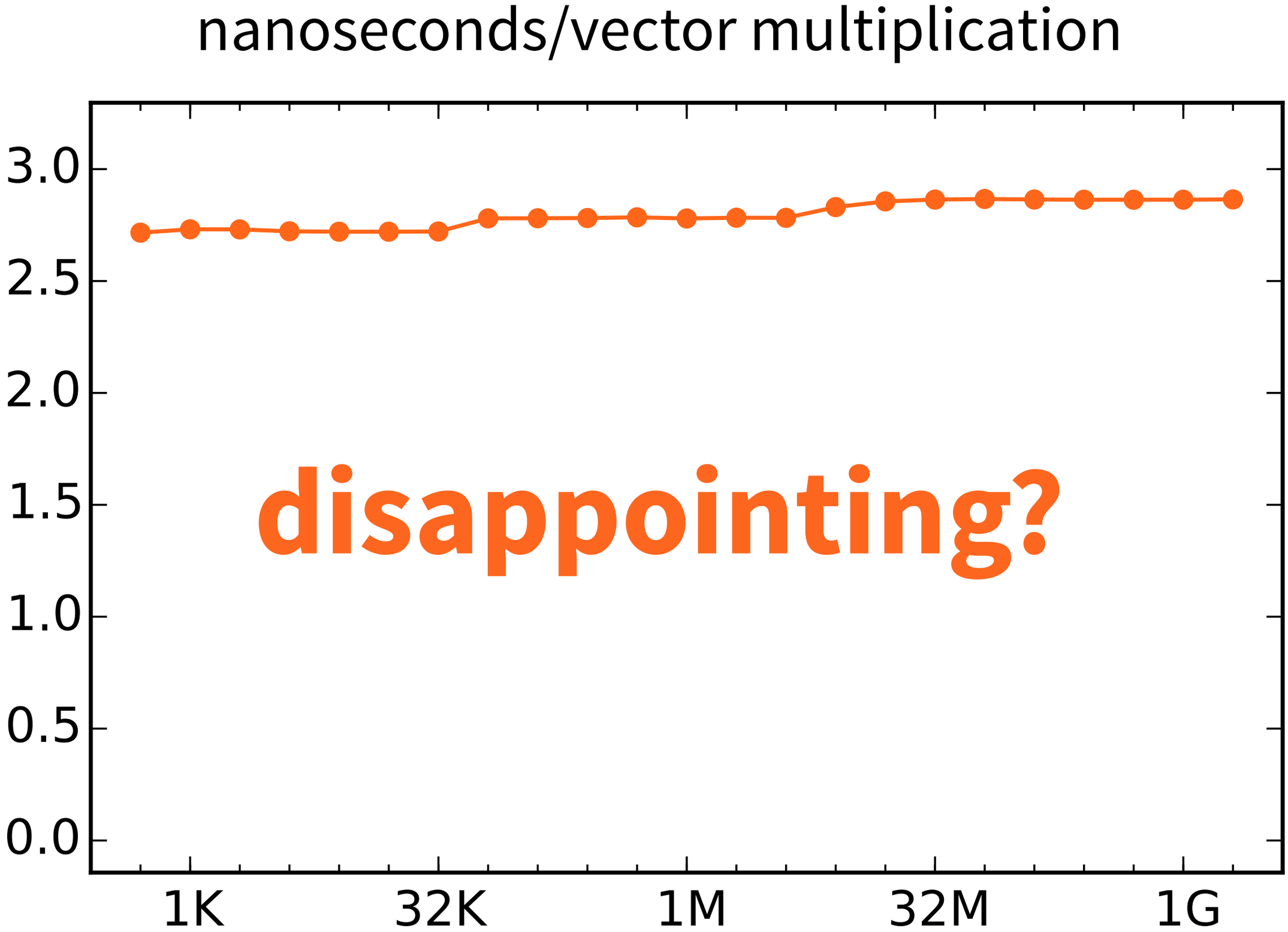
— memory hierarchy

Toy example: sum of squares

```
// Repeatedly do multiply-and-add
// for an array of with “size” vectors
for (int j = 0; j < iter; ++j) {
    for (int i = 0; i < size; ++i) {
        double4_t y = v[i];
        x += y * y;
    }
}
```

How well does this perform?


nanoseconds/vector multiplication
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1K
32K
1M
32M
1G
array size in bytes


nanoseconds/vector multiplication
3.0
2.5
2.0
1.5
1.0
0.5
0.0
disappointing?
1K
32K
1M
32M
1G
array size in bytes

Instruction-level parallelism

- **Good: parallelism in vector operations**
- **Bad: very little opportunities for instruction-level parallelism**
- **Inherently sequential:**

```
x += y[0]*y[0]; x += y[1]*y[1];
x += y[2]*y[2]; x += y[3]*y[3]; ...
```

Bad

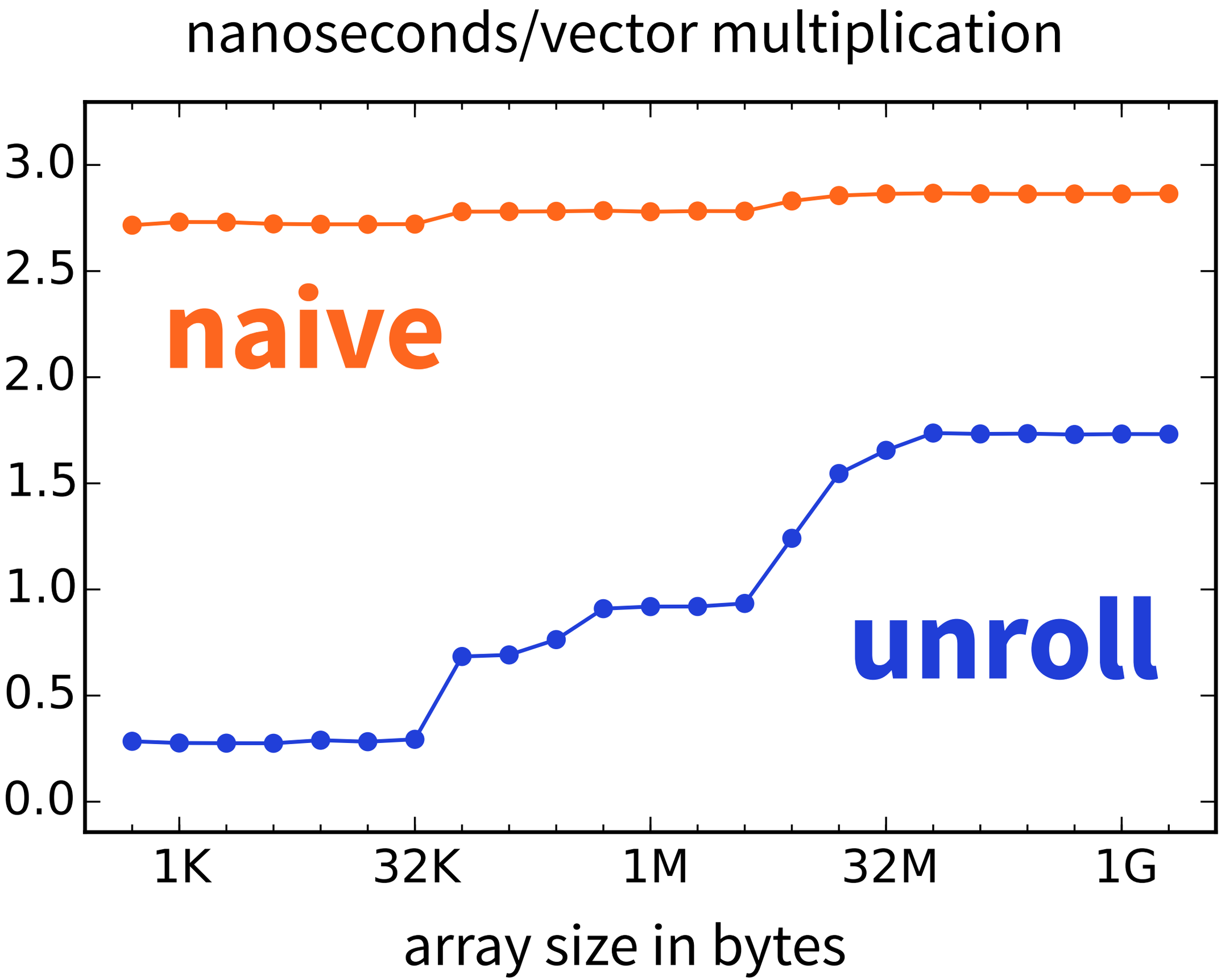
```
x += y[0]*y[0];
x += y[1]*y[1];
x += y[2]*y[2];
x += y[3]*y[3];
x += y[4]*y[4];
x += y[5]*y[5];
x += y[6]*y[6];
x += y[7]*y[7];
x += y[8]*y[8];
...
```

Better

```
t[0] += y[0]*y[0];
t[1] += y[1]*y[1];
t[2] += y[2]*y[2];
t[3] += y[3]*y[3];
t[0] += y[4]*y[4];
t[1] += y[5]*y[5];
t[2] += y[6]*y[6];
t[3] += y[7]*y[7];
t[0] += y[8]*y[8];
...
```

```
//  More opportunities for instruction-level parallelism
//  (assuming here that “size” is a multiple of 8)
double4_t t[8];
...
for (int j = 0; j < iter; ++j) {
    for (int i = 0; i < size; i += 8) {
        for (int k = 0; k < 8; ++k) {
            double4_t y = v[i + k];
            t[k] += y * y;
        }
    }
}
for (int k = 0; k < 8; ++k) {
    x += t[k];
}
```

Any improvements?

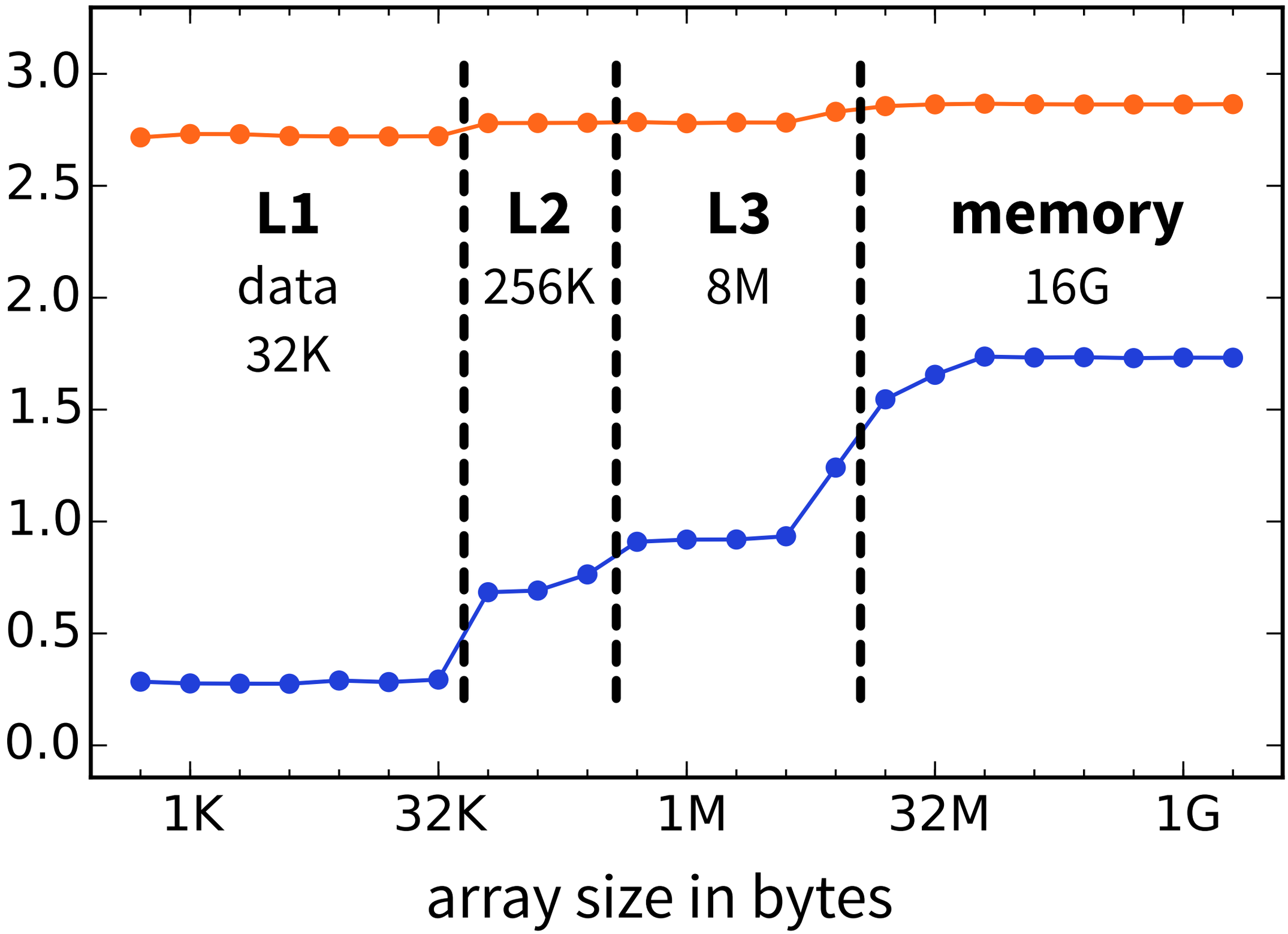

nanoseconds/vector multiplication
3.0
2.5
2.0
1.5
1.0
0.5
0.0
naive
unroll
1K
32K
1M
32M
1G
array size in bytes

Bottlenecks

- **Naive version:**
  latency of vector operations

- **Unrolled version, small data:**
  throughput of vector operations

- **Unrolled version, large data:**
  getting data from the ***memory***


nanoseconds/vector multiplication
3.0
2.5
2.0
1.5
1.0
0.5
0.0
L1
data
32K
L2
256K
L3
8M
memory
16G
1K
32K
1M
32M
1G
array size in bytes

Caches

How do caches work?

- **CPU ↔ L1 ↔ L2 ↔ L3 ↔ memory**
- **Whenever you read memory:**
  - CPU reads the full ***cache line*** (**64 bytes**) from the nearest cache that contains it
  - stores it in all intermediate caches, makes room by throwing away older data

Some rules of thumb

- **Repeatedly work with a small chunk of << 32KB of data:**
  - all data remains in L1
  - small latency (order of 1 ns)
  - large bandwidth

Some rules of thumb

- **Random reads in >> 8MB of data:**
  - most memory lookups are cache misses
  - large latency (order of 100 ns)
  - small bandwidth

Some rules of thumb

- **Ideal:** linear scanning of L1
- **Good:** random access of L1,
  linear scanning of L2–L3
- **Tolerable:** linear scanning of main memory
- **Horrible:** random access of main memory

nanoseconds/vector multiplication

10^1

10^0

non-linear read

naive

unroll

linear read

1K 32K 1M 32M 1G

array size in bytes

Some rules of thumb

- **You can do useful work while you wait for data from memory**

- ***Instruction-level parallelism*** **does it automatically, if there are some other *independent* operations that you can run**

Optimising cache usage

cache blocking in matrix multiplication

Arithmetic intensity

- **Throughput of arithmetic operations larger than main memory bandwidth**

- **Whenever you read data from main memory to caches** (or from caches to registers)**, try to do *many arithmetic operations with the same data***

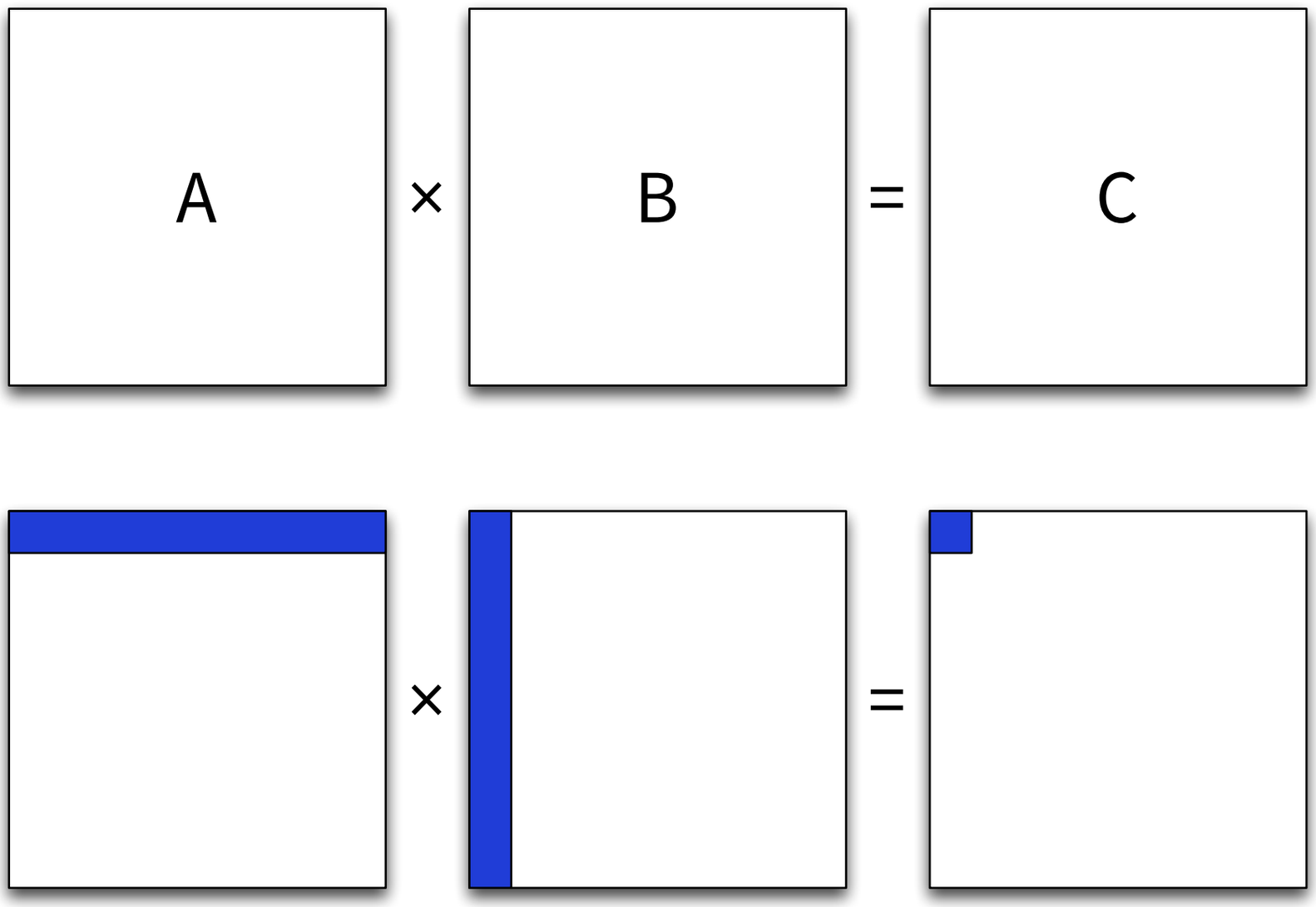
Example: matrix multiplication

- **Multiplying $n \times n$ matrixes:**
  $O(n^2)$ data, $O(n^3)$ operations

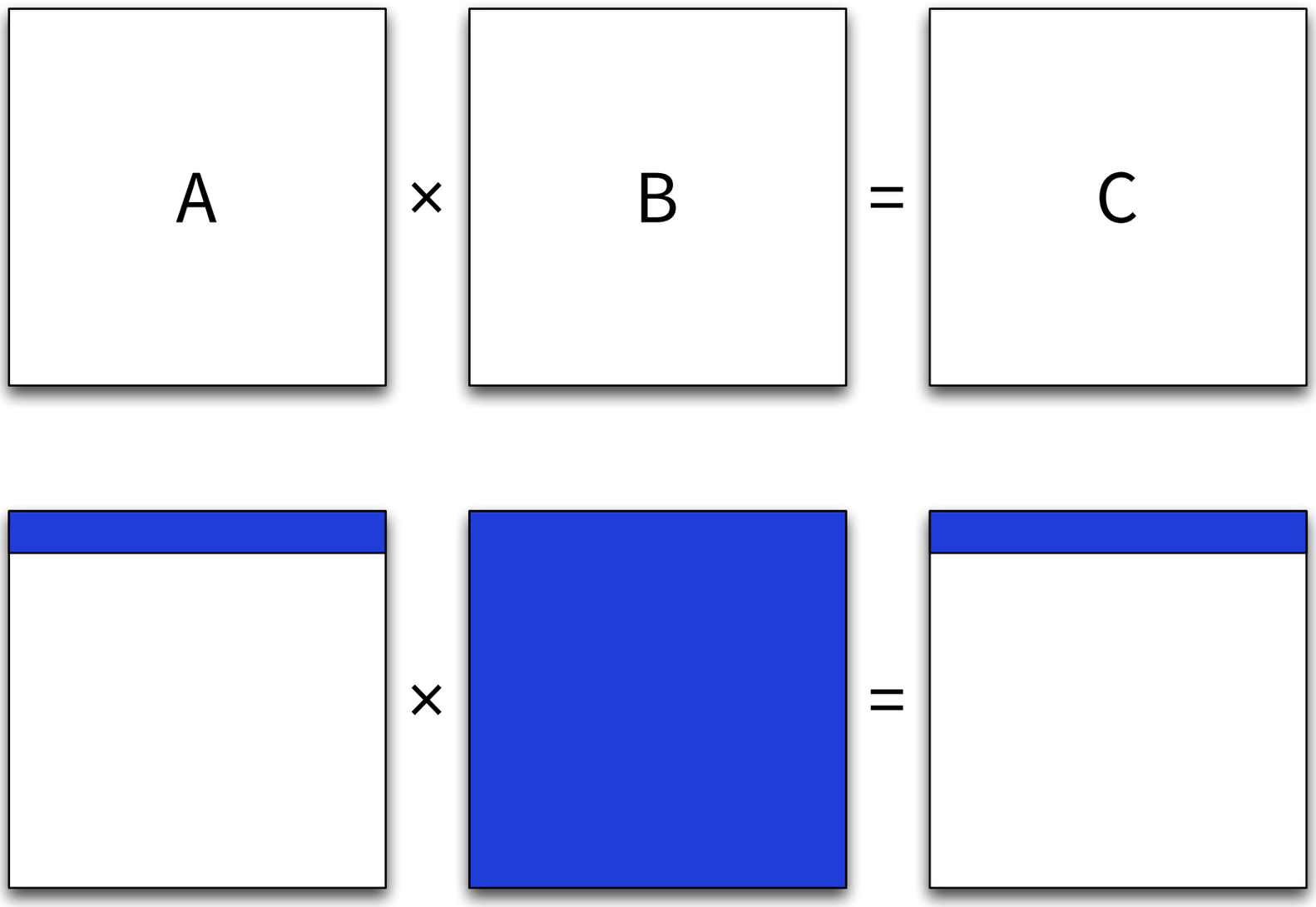
- **Naive algorithm: each operation needs to *fetch new data from memory***

- **Better algorithm: most operations *use data that is already in cache or registers***

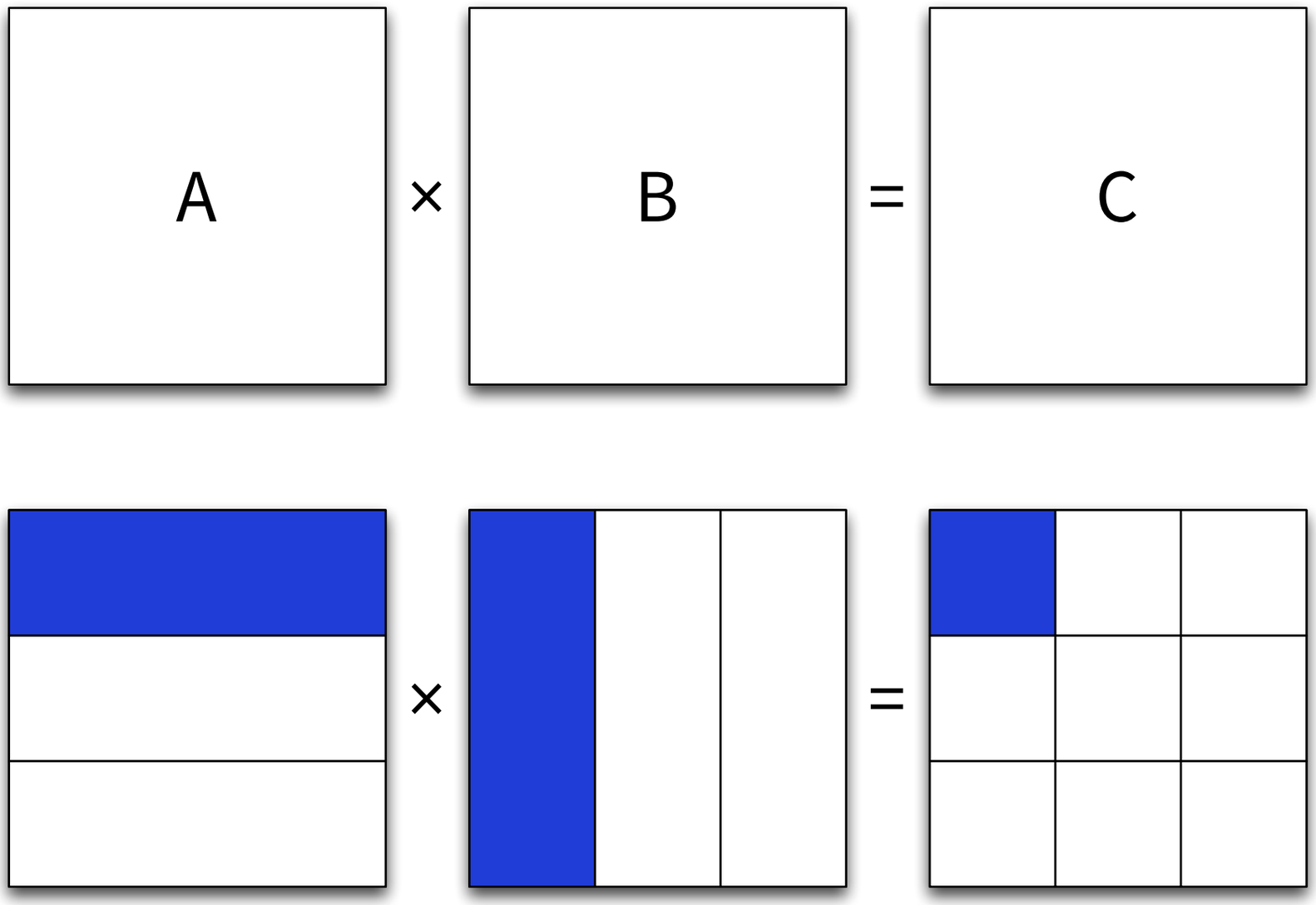
Matrix multiplication



Naive solution: poor locality



Cache blocking: better locality

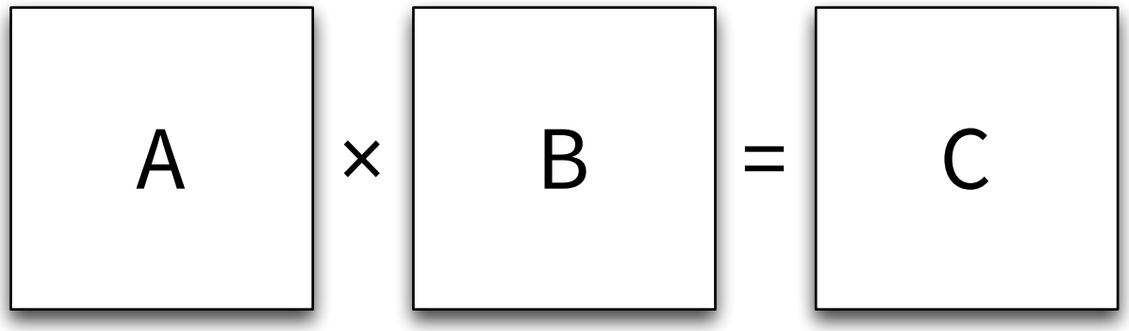


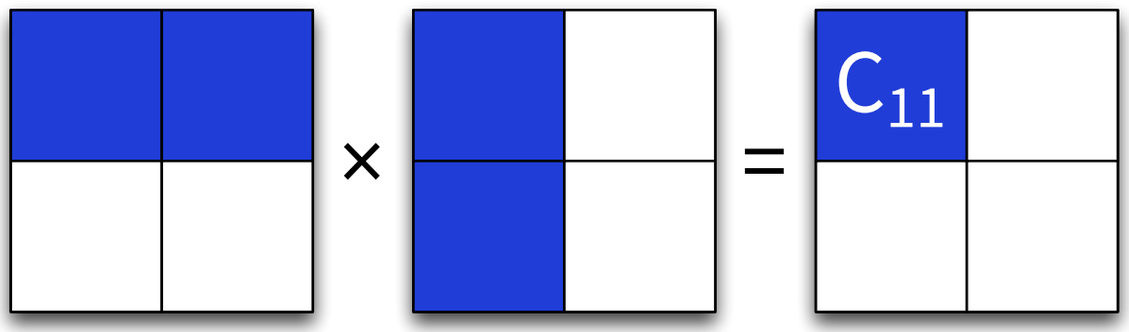
Reusing data in registers

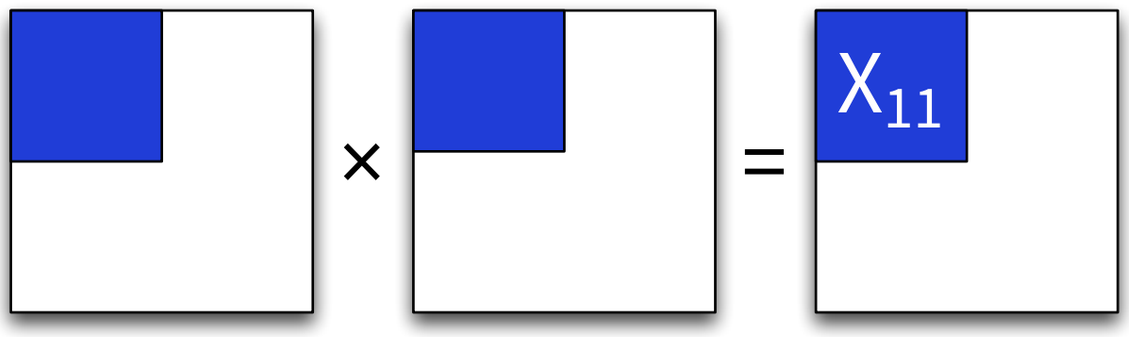
- **Naive: calculate 1 dot product $x_1 \cdot y_1$**
- **Better: calculate simultaneously 4 dot products $x_1 \cdot y_1$, $x_1 \cdot y_2$, $x_2 \cdot y_1$, $x_2 \cdot y_2$**
  - read 2 times as much data
  - produce 4 times as many results
  - better *arithmetic intensity*

Reusing data in registers

- **Naive: calculate 1 dot product $x_1 \cdot y_1$**
- **Better: calculate simultaneously 9 dot products $x_1 \cdot y_1$, …, $x_3 \cdot y_3$**
  - read 3 times as much data
  - produce 9 times as many results
  - still enough registers to keep everything…?







$$C_{11} = X_{11} + Y_{11}$$

Summary

- **Use *vector instructions* to better exploit parallel processing units in modern CPUs**
- **Pay attention to *caches*: reuse data**
- **Do not forget *instruction-level parallelism***
- **Do not forget *using multiple threads***