Jukka Suomela
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What really happened in Helsinki and can you mend it?

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Two memories:

What really happened in Helsinki?

SIROCCO 2016

Can you mend it?

SIROCCO 2022

What really happened in Helsinki?

SIROCCO 2016

SIROCCO 2016

Helsinki, Finland

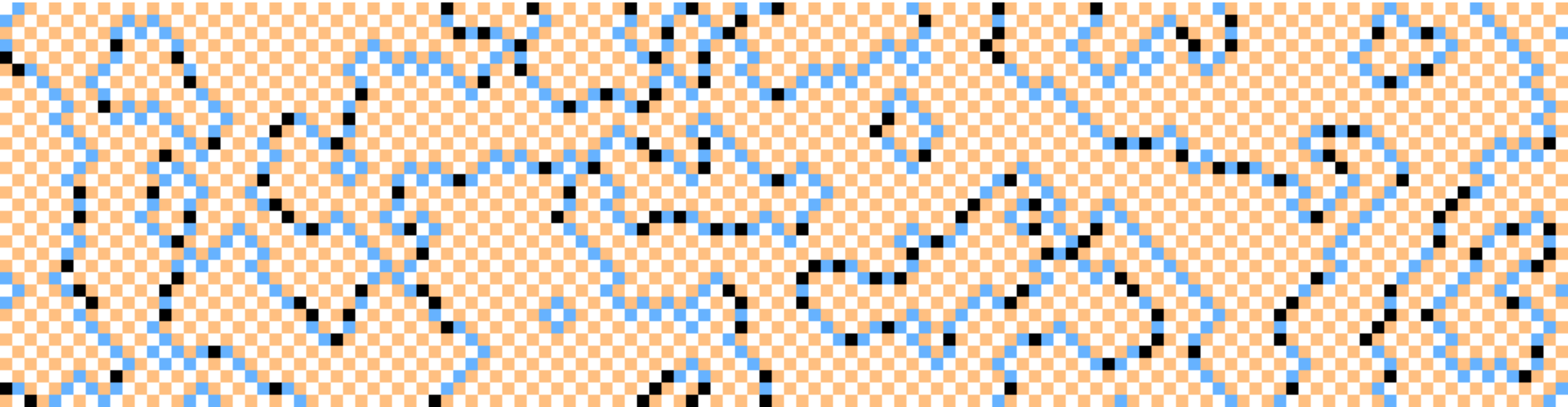
Local organizers:

Jukka Suomela
Christopher Purcell
Juho Hirvonen
Tuomo Lempiäinen
Joel Rybicki

Can you mend it?

Around 2015, we started to study a seemingly simple question:

Can you 4-color a grid locally?



Around 2015, we started to study a seemingly simple question:

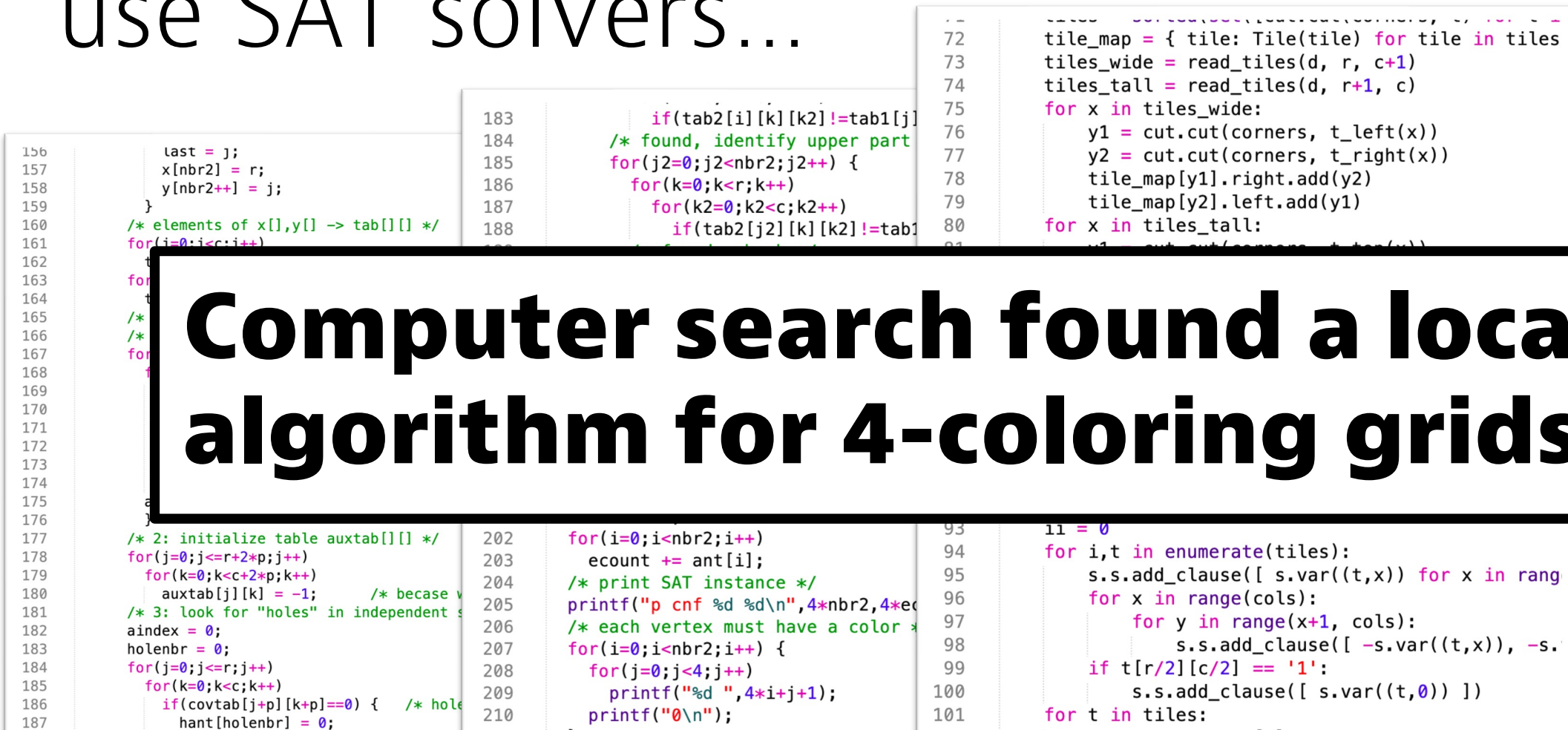
Can you 4-color a grid locally?

- 2-coloring: trivially hard
- 3-coloring: not-so-trivially hard
- 4-coloring: **???**
- 5-coloring: easy (Δ+1 colors)

We tried the usual strategy:
sit in front of a whiteboard
and think hard…

Zero progress

We tried an usual strategy: write lots of C and Python code, use SAT solvers…



```
156             last = j;
157             x[nbr2] = r;
158             y[nbr2++] = j;
159           }
160         /* elements of x[],y[] -> tab[][] */
161         for(j=0;j<c;j++)
162           t
163         for
164           t
165         /*
166         /*
167         for
168           f
169
170
171
172
173
174
175           a
176           }
177         /* 2: initialize table auxtab[][] */
178         for(j=0;j<=r+2*p;j++)
179           for(k=0;k<c+2*p;k++)
180             auxtab[j][k] = -1;      /* becase w
181         /* 3: look for "holes" in independent s
182         aindex = 0;
183         holenbr = 0;
184         for(j=0;j<=r;j++)
185           for(k=0;k<c;k++)
186             if(covtab[j+p][k+p]==0) {   /* hole
187               hant[holenbr] = 0;
```

```
183             if(tab2[i][k][k2]!=tab1[j]
184           /* found, identify upper part
185           for(j2=0;j2<nbr2;j2++) {
186             for(k=0;k<r;k++)
187               for(k2=0;k2<c;k2++)
188                 if(tab2[j2][k][k2]!=tab1
202       for(i=0;i<nbr2;i++)
203         ecount += ant[i];
204       /* print SAT instance */
205       printf("p cnf %d %d\n",4*nbr2,4*ec
206       /* each vertex must have a color *
207       for(i=0;i<nbr2;i++) {
208         for(j=0;j<4;j++)
209           printf("%d ",4*i+j+1);
210         printf("0\n");
```

```
72      tile_map = { tile: Tile(tile) for tile in tiles
73      tiles_wide = read_tiles(d, r, c+1)
74      tiles_tall = read_tiles(d, r+1, c)
75      for x in tiles_wide:
76          y1 = cut.cut(corners, t_left(x))
77          y2 = cut.cut(corners, t_right(x))
78          tile_map[y1].right.add(y2)
79          tile_map[y2].left.add(y1)
80      for x in tiles_tall:
93      ii = 0
94      for i,t in enumerate(tiles):
95          s.s.add_clause([ s.var((t,x)) for x in rang
96          for x in range(cols):
97              for y in range(x+1, cols):
98                  s.s.add_clause([ -s.var((t,x)), -s.
99          if t[r/2][c/2] == '1':
100             s.s.add_clause([ s.var((t,0)) ])
101     for t in tiles:
```

Computer search found a local algorithm for 4-coloring grids!

Can you 4-color a grid locally?

- 2-coloring: trivially hard
- 3-coloring: not-so-trivially hard
- 4-coloring: **yes, but it's complicated**
- 5-coloring: easy (Δ+1 colors)

PODC 2017

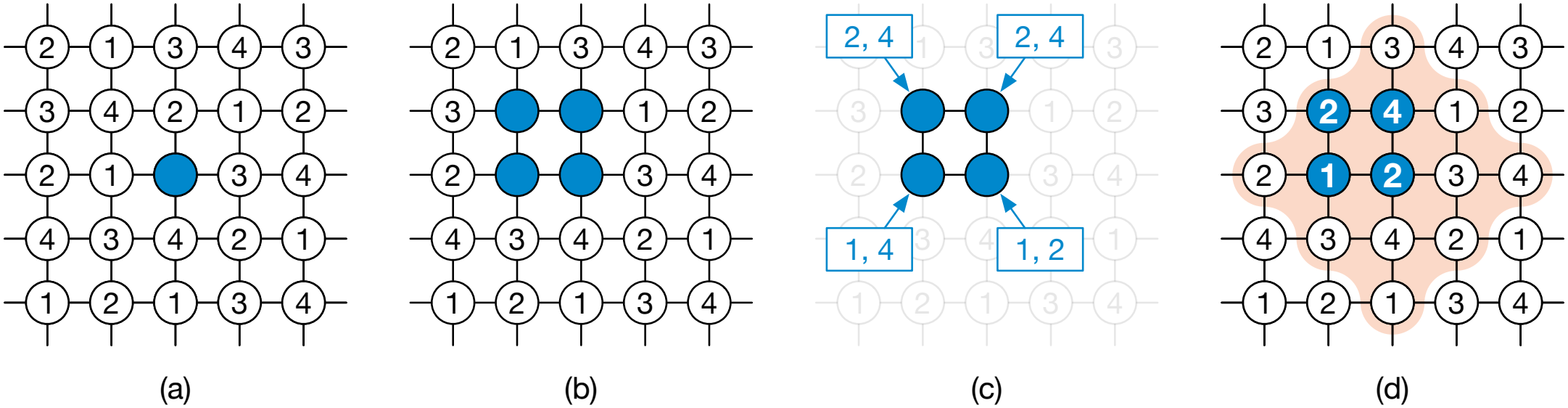
Brandt, Hirvonen, Korhonen, Lempiäinen, Östergård, Purcell, Rybicki, S., Uznański: "LCL problems on grids"

Much later…

We realized it doesn’t need to be complicated!

You just need the right perspective:
local mending

Local mending for 4-colorings



Simple special case of a more general result
by Chechik and Mukhtar (SODA 2019)

Local mending: generalization of greedily completable problems

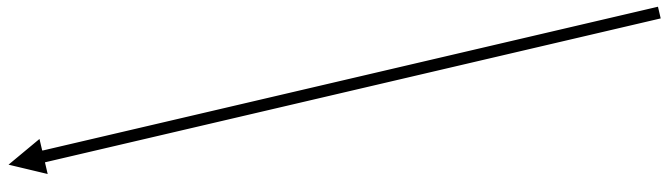
Mending radius: how far do you need to undo a partial solution to "patch a hole"?

SIROCCO 2022

Balliu, Hirvonen, Melnyk, Olivetti, Rybicki, S.:
"Local mending"

An example:
LCLs (locally checkable labelings) in trees

Solving	**Mending**
$O(1)$	$O(1)$
$O(\log^* n)$	
$O(\log n)$	$O(\log n)$
$O(n^{1/k})$	$O(n)$



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