**Data Structures and Algorithms 2, Fall 1998**
Problem Set 5 (7 – 9 Dec)

1. Determine whether the following propositional formulas are satisfiable:

   (a) $\neg x_1 \wedge x_3 \wedge (\neg x_2 \vee \neg x_3)$;

   (b) $\bigwedge_{ijk}(x_i \vee x_j \vee x_k) \wedge \bigwedge_{ijk}(\neg x_i \vee \neg x_j \vee \neg x_k)$.

   In formula (b), the indexing is over all triples $(i, j, k)$ such that $i, j, k \in \{1, \ldots, 5\}$ and $i \neq j$, $j \neq k$, $k \neq i$.

2. Suppose that SAT $\in$ P. Show that there then exists a polynomial time algorithm that for each formula $F(x_1, x_2, \ldots, x_n)$ either states that the formula is not satisfiable, or actually finds a satisfying truth assignment to the variables $x_1, x_2, \ldots, x_n$. (*Hint:* It is easy to see that formula $F(x_1, x_2, \ldots, x_n)$ is satisfiable, if and only if either formula $F(0, x_2, \ldots, x_n)$ or formula $F(1, x_2, \ldots, x_n)$ is satisfiable.)

3. Prove the following Lemmas (p. 115 of the lecture notes):

   (a) If $L$ is an NP-complete set and $L \in$ P, then P = NP.

   (b) If $L$ is an NP-complete set, $L' \in$ NP and $L \leq_m^p L'$, then also $L'$ is NP-complete.

4. Construct a polynomial time reduction from the *Clique* problem to the *Vertex Cover* (VC) problem. (*Hint:* Look at the reductions presented on pp. 111–112 of the lecture notes.)

5. Prove that the following *Set Packing* problem is NP-complete:

   > Given a collection of subsets $C_1, \ldots, C_n$ of a finite set $U$, and a natural number $k$, is it possible to choose a subcollection of at least $k$ *disjoint* $C_i$'s (i.e. $C_i \cap C_j = \emptyset$ for $i \neq j$)?

   (*Hint:* Reduction from the *Independent Set (IS)* problem.)

**Reminder: This is the last problem set. The last lecture of the course is on Monday, 7 Dec, and the course exam takes place on Tuesday, 15 Dec (8–12).**