Data Structures and Algorithms 2, Fall 1998

Problem Set 1 (9–11 Nov.)

1. Arrange the following functions in an increasing sequence according to their order of growth (big-O) classes:

$$n, \quad \log n, \quad \log^2 n, \quad \sqrt{n}\log^2 n, \quad (\tfrac{3}{2})^n,$$
$$\sqrt{n}, \quad \log\log n, \quad \tfrac{n}{\log n}, \quad (\tfrac{1}{3})^n, \quad 17.$$

(The notation "$\log n$" denotes here and in the problems below base-2 logarithms.)

2. Show that

$$\log(n!) = \sum_{i=1}^{n} \log i = \Theta(n \log n).$$

(*Hint:* Construct an upper bound for the value of the sum using the $\log n$ term, and a lower bound using the term $\log \frac{n}{2}$.)

3. Analyze the worst-case complexity of the following "selection sort" algorithm:

$$
\begin{array}{l}
\textit{selectsort}(A[1\ldots n]):\\
\quad \textbf{for } i \leftarrow 1 \textbf{ to } n-1 \textbf{ do}\\
\quad \{\ \ \textit{amin} \leftarrow A[i];\ \textit{imin} \leftarrow i;\\
\qquad \textbf{for } j \leftarrow i+1 \textbf{ to } n \textbf{ do}\\
\qquad\quad \textbf{if } A[j] < \textit{amin} \textbf{ then}\\
\qquad\qquad \{\textit{amin} \leftarrow A[j];\ \textit{imin} \leftarrow j\}\\
\qquad A[\textit{imin}] \leftarrow A[i];\ A[i] \leftarrow \textit{amin}\\
\quad \}
\end{array}
$$

4. Solve, by unwinding, the following recurrence equation:

$$\begin{cases} T(0) &= 1, \\ T(n) &= 2T(n-1)+1, \quad \text{when } n \geq 1. \end{cases}$$

5. Determine the order of growth (big-O) classes for the following recurrence equations. In each case the boundary condition is $T(1) = 1$, and the functions need to be considered only for values of n that are powers of 2. (*Note:* Item (d) is slightly more difficult than the others.)

   (a) $T(n) = T(n/2) + 1$,
   (b) $T(n) = 2T(n/2) + n$,
   (c) $T(n) = 2T(n/2) + n^2$.
   (d) $T(n) = 2T(n/2) + \log n$,

6. Analyze the worst-case complexity of the following recursive algorithm for determining the maximum and minimum elements of an array $A[1\ldots n]$, in the case when n is a power of 2.

$$
\begin{array}{l}
\textbf{function } \textit{maxmin}(A[1\ldots n]);\\
\textbf{begin}\\
\quad \textbf{if } n = 1 \textbf{ then return } (A[1], A[1])\\
\quad \textbf{else begin}\\
\qquad (\textit{max}1, \textit{min}1) := \textit{maxmin}(A[1\ldots \lfloor n/2 \rfloor]);\\
\qquad (\textit{max}2, \textit{min}2) := \textit{maxmin}(A[\lfloor n/2 \rfloor + 1 \ldots n]);\\
\qquad \textbf{return } (\max\{\textit{max}1, \textit{max}2\}, \min\{\textit{min}1, \textit{min}2\})\\
\quad \textbf{end}\\
\textbf{end } \{\textit{maxmin}\}.
\end{array}
$$