Aalto University
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Mining temporal networks

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Workshop on online social networks and media
Network properties and dynamics (ONSED)

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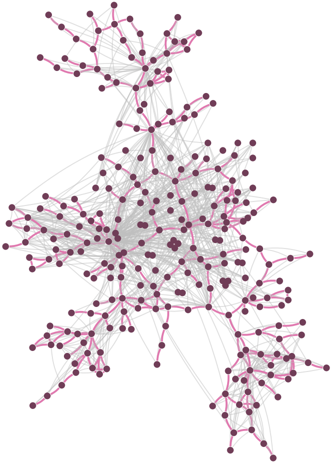
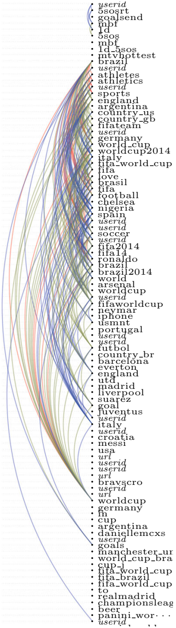
interconnected world

- networks model objects and their relations
- many different network types
  - social
  - informational
  - technological
  - biological
  - …



impact of network science

- online communication networks and social media
- implications in
  - knowledge creation
  - information sharing
  - education
  - democracy
  - society as a whole



research questions

- structure discovery
  - finding communities, events, roles of individuals
- study complex dynamic phenomena
  - evolution, information diffusion, opinion formation
- develop novel applications
- design efficient algorithms

traditional view

- networks represented as pure graph-theory objects

  no additional vertex / edge information

- emphasis on static networks
- dynamic settings model structural changes

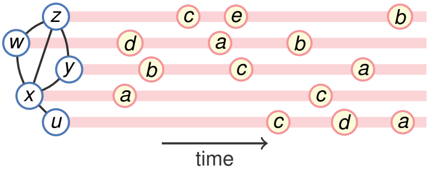
  vertex / edge additions / deletions

temporal networks

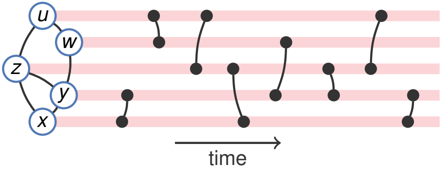
- ability to collect and store large volumes of network data
- available data have fine granularity
- lots of additional information associated to vertices/edges
- network topology is relatively stable, while
  lots of activity and interaction is taking place
- giving rise to new concepts, new problems, and
  new computational challenges

modeling activity in networks

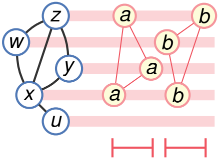
1. network nodes perform actions (e.g., posting messages)



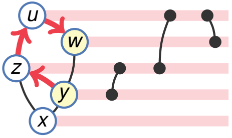
2. network nodes interact with each other
(e.g., a “like”, a repost, or sending a message to each other)



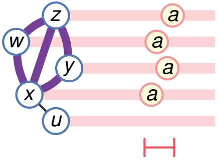
many novel and interesting concepts



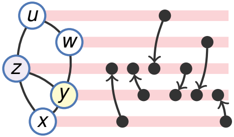
new pattern types



temporal information paths



new types of events



network evolution

temporal networks — objectives

- identify new concepts and new problems
- develop algorithmic solutions
- demonstrate revelance to real-world applications

agenda

tracking important nodes

- temporal PageRank
- maintaining neighborhood profiles

reconstruction problems

- reconstructing an epidemic over time
- reconstruction of activity timelines

tracking important nodes

temporal PageRank

P. Rozenshtein and A. Gionis, ECML PKDD 2016

PageRank

- classic approach for measuring node importance
- listed in the top-10 most important data-mining algorithms

  [Wu et al., 2008]
- numerous applications
  - ranking web pages
  - trust and distrust computation
  - finding experts in social networks
  - …

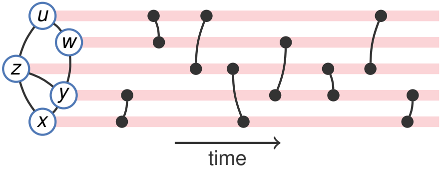
PageRank

- PageRank defined as the stationary distribution of a random walk in the graph
- inherently a static process
- however, many modern networks can be viewed as a sequence (stream) of edges
  - temporal network : $G = (V, E)$, with $E = \{(u, v, t)\}$
  - examples : twitter, instagram, IMs, email, ...
- what is an appropriate PageRank definition for temporal networks?

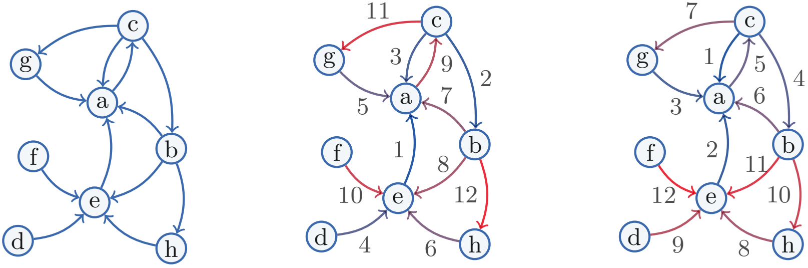
temporal networks

network nodes interact with each other

(e.g., a "like", a repost, or sending a message to each other)



motivating example



(a) static network

(b) temporal network

(c) temporal network

research questions and objectives

- extend PageRank to incorporate temporal information and network dynamics
- adapt PageRank to reflect changes in network dynamics and node importance
- estimate importance of a node u at any given time t

dynamic PageRank vs. temporal PageRank

- extensive work on dynamic PageRank
- dynamic PageRank computation :
  maintain correct PageRank during network updates
  - e.g., edge additions / deletions
- computation should return the static PageRank at a given network snapshot
- for edges present in a snapshot, order does not matter

static PageRank

- graph $G = (V, E)$
- corresponding row-stochastic matrix $P \in \mathbb{R}^{n \times n}$
- personalization vector $\mathbf{h} \in \mathbb{R}^n$
- PageRank is the stationary distribution of a random walk, with restart probability $(1 - \alpha)$

$$\pi(u) = \sum_{v \in V} \sum_{k=0}^{\infty} (1-\alpha)\alpha^k \sum_{\substack{z \in \mathcal{Z}(v,u) \\ |z|=k}} h(v) \Pr[z \mid v]$$

where, $\mathcal{Z}(v, u)$ is the set of all paths from v to u

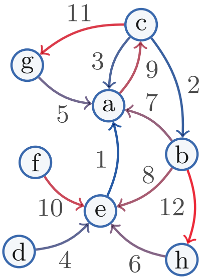
and $\Pr[z \mid v] = \prod_{(i,j) \in z} P(i, j)$

temporal PageRank

- make a random walk only on temporal paths

  e.g., time-respecting paths

  time-stamps increase along the path



$c \to b \to a \to c$: time respecting

$a \to c \to b \to a$: not time respecting

temporal PageRank

- intuition : probability of visiting node u at time t
  given a random walk on temporal paths
- need to model probability of following next temporal edge
  - we use an exponential distribution
- temporal PageRank definition

$$r(u, t) = \sum_{v \in V} \sum_{k=0}^{t} (1-\alpha)\alpha^k \sum_{\substack{z \in \mathcal{Z}^T(v,u|t) \\ |z|=k}} \Pr{}'[z \mid t]$$

$\mathcal{Z}^T(v, u \mid t)$ set of temporal paths from v to u until time t

computation

- simple online algorithm
- $r(u, t)$: temporal PageRank estimate of u at time t
- $s(u, t)$: count of active walks visiting u at time t

input : E, transition probability β, jumping probability α

$\boldsymbol{r} = \mathbf{0}$, $\boldsymbol{s} = \mathbf{0}$;

foreach $(u, v, t) \in E$ **do**

  $\boldsymbol{r}(u) = \boldsymbol{r}(u) + (1 - \alpha)$;

  $\boldsymbol{r}(v) = \boldsymbol{r}(v) + (\boldsymbol{s}(u) + (1 - \alpha))\alpha$;

  $\boldsymbol{s}(v) = \boldsymbol{s}(v) + (\boldsymbol{s}(u) + (1 - \alpha))(1 - \beta)\alpha$;

  $\boldsymbol{s}(u) = (\boldsymbol{s}(u) + (1 - \alpha))\beta$;

normalize $\boldsymbol{r}$;

return $\boldsymbol{r}$;

static vs. temporal PageRank

- temporal PageRank is designed to capture changes in network dynamics and concept drifts
- what if the edge distribution is stable?

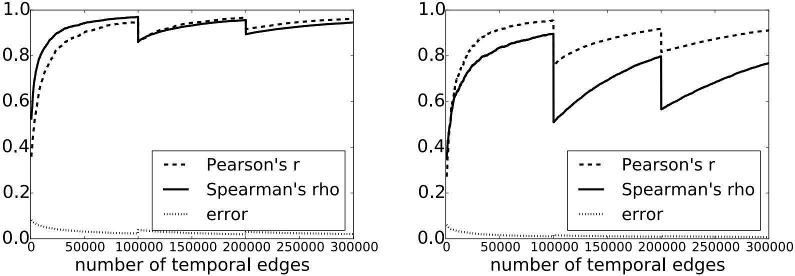
static vs. temporal PageRank

- consider static network $G_S = (V, E_S, w)$
- time period $[1, \ldots, T]$
- construct temporal network $G = (V, E)$ by sampling edges proportionally to their weight

proposition :

as $T \to \infty$, the temporal PageRank on G
converges to the static PageRank on G_S,
with personalization vector equal to weighted out-degree

experiment — adaptation to concept drift



(a) *Facebook* (b) *Twitter*

tracking important nodes

maintaining sliding-window neighborhood profiles

R. Kumar, T. Calders, A. Gionis, and N. Tatti, ECML PKDD 2015

distance distributions in graphs

- given graph G, a node u, and distance r :

  how many nodes of G are in distance r from u?

- fundamental graph-mining primitive
  - median distance, diameter, effective diameter
- related to small-world phenomena
- a measure of centrality for nodes of G

distance distributions in graphs

- exact solution requires all-pairs shortest path computation
  - Floyd-Warshall algorithm: $\mathcal{O}(n^3)$
  - or, BFS for unweighted graphs: $\mathcal{O}(nm)$
- clearly non scalable
- resort to approximations based on diffusion methods

diffusion-based computation

[Palmer et al., 2002]

- let $B_t(x)$ be the ball of radius t around x (the set of nodes at distance $\leq t$ from x)
- clearly $B_0(x) = \{x\}$
- moreover $B_{t+1}(x) = \bigcup_{(x,y)} B_t(y) \bigcup \{x\}$
- so computing B_{t+1} from B_t just takes a single (sequential) scan of the graph

diffusion-based computation

- every set requires $O(n)$ bits, hence $O(n^2)$ bits overall
- amount of space is prohibitively large
- instead use sketching for counting distinct elements
- probabilistic counters require very small space ($\log \log$)
- HyperANF algorithm [Boldi et al., 2011]
  - uses HyperLogLog counters [Flajolet et al., 2007]
  - with 40 bits you can count up to 4 billion with standard deviation 6%

Separating You and Me? 4.74 Degrees

By JOHN MARKOFF and SOMINI SENGUPTA
Published: November 21, 2011

RECOMMEND
TWITTER
LINKEDIN
SIGN IN TO E-MAIL
PRINT
REPRINTS
SHARE

The world is even smaller than you thought.

Enlarge This Image

Cornell News Service

Jon Kleinberg of Cornell said weak ties could be important.

Adding a new chapter to the research that cemented the phrase "six degrees of separation" into the language, scientists at Facebook and the University of Milan reported on Monday that the average number of acquaintances separating any two people in the world was not six but 4.74.

The original "six degrees" finding, published in 1967 by the psychologist Stanley Milgram, was drawn from 296 volunteers who were asked to send a message by postcard, through friends and then friends of friends, to a specific person in a Boston suburb.

Log in to see what your friends are sharing on nytimes.com. Privacy Policy | What's This?
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What's Popular Now

Marvin Hamlisch, Composer, Dies at 68

France's 'les Riches' Vow to Leave if 75% Tax Rate Is Passed

extension to temporal networks

- limitations of existing solutions
  - consider static network
  - multi-pass algorithm
- in this work
  - extension to temporal networks
  - streaming algorithm for sliding-window model :

    consider only the most recent interactions (edges)

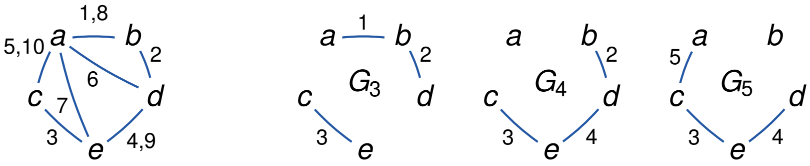
setting

- temporal network $G = (V, E)$
- stream of edges $E = \langle (u_1, v_1, t_1), (u_2, v_2, t_2), \ldots \rangle$
  with $t_1 \leq t_2 \leq \ldots$
- sliding window length w
- snapshot network $G(t, w)$ at time t contains all edges
  with time-stamps in $(t - w, t]$

problem :

given node u, window length w, and distance r, how many nodes in $G(t, w)$ are within distance r from u at time t?

example



a toy example, 3 snapshot graphs with a window size of 3

proposed online algorithms

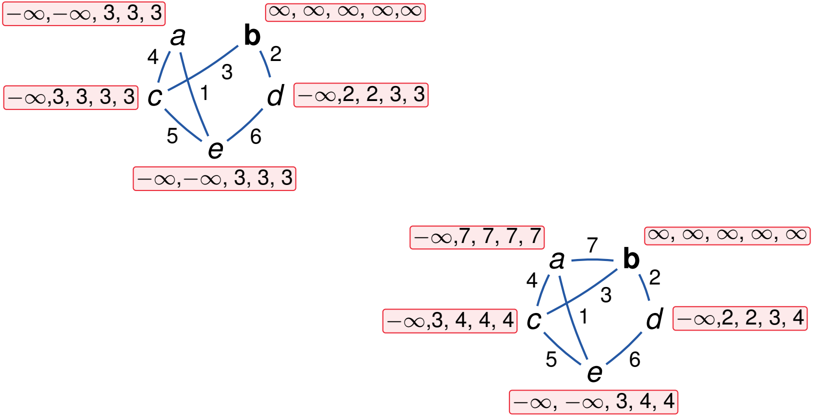
1. an exact but memory-inefficient streaming algorithm
2. an approximate memory-efficient streaming algorithm

– approximate algorithm uses logic of exact algorithm, combined with hyperloglog sketches

horizons

- path horizon : time-stamp of the oldest edge on the path
- $h(u, v, i)$: the horizon for length i between nodes u and v : the maximum horizon of any path of length at most i

example



two snapshot graphs along with $h(u, b, i)$ for $i = 0, \ldots, 4$

neighborhood summaries

- observation : if for a node u we know all horizons $h(u, v, i)$, for all distances i and all nodes v, we can give complete neighborhood profile for u for any window length
- neighborhood summary : $S_t^u = (S_t^u[0], \ldots, S_t^u[r])$
  where $S_t^u[i] = \{(v, h_t(u, v, i)) \mid h_t(u, v, i) > -\infty\}$

updating neighborhood summaries

- edge deletion : simply delete entries from summaries
- edge addition : a change in summary at distance i for a node u will introduce a change in the summary of its neighbors at distance $i + 1$
  - updates propagate in a BFS fashion

exact algorithm

- update time : $\mathcal{O}(rmn \log n)$
- space complexity : $\mathcal{O}(rn^2)$

  where r an upper bound on max distance

- quadratic dependence not acceptable for large graphs
  - hence approximation algorithm

approximate algorithm

- sliding HyperLogLog sketch : extension of HyperLogLog to maintain a distinct set counter over sliding window
- if number of buckets in the HLL counter is k then the worst case complexity changes to

– update time :

$$\mathcal{O}(rm2^k \log^2 n) \qquad \text{from} \qquad \mathcal{O}(rmn \log n)$$

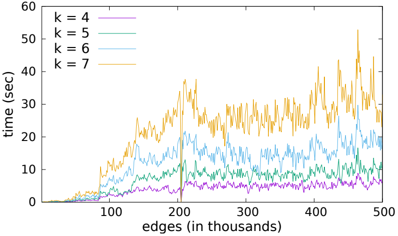
– space complexity :

$$\mathcal{O}(rn2^k \log n) \qquad \text{from} \qquad \mathcal{O}(rn^2)$$

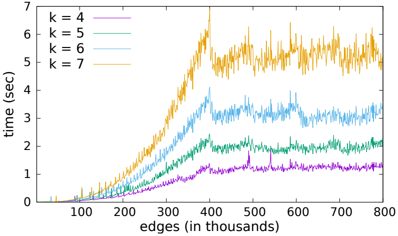
empirical evaluation — quality

dataset	nodes	dist edges	total edges	clus coef	diam	eff diam	avg rel error (k=7)
Facebook	4 039	88 234	88 234	0.60	8	4.7	0.08
Cit-HepTh	27 771	352 801	352 801	0.31	13	5.3	0.10
Higgs	166 840	249 030	500 000	0.19	10	4.7	0.14
DBLP	192 357	400 000	800 000	0.63	21	8.0	0.09

empirical evaluation — running time



(c) Higgs



(d) DBLP

contrast (DBLP)

– offline HyperANF : 3.6 sec / sliding window
– proposed approach : 0.003 sec / sliding window

reconstructing an epidemic over time

P. Rozenshtein, A. Gionis, B.A. Prakash, J. Vreeken, KDD 2016

motivation

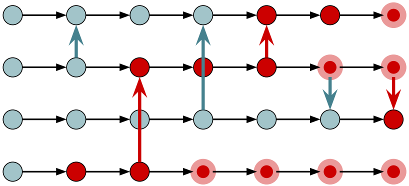
- consider a sequence of timestamped edges
  - an edge between people represents some interaction
    phonecall, email, retweet, ...
- infection reconstruction :
  - consider a unknown dynamic propagation process
    virus, idea, topic, gossip, ...
  - incomplete reported cases of infection
- goal :
  - reconstruct paths of infection,
    which explains cases of reported infection, and
    recovers missing infected nodes and interactions

model

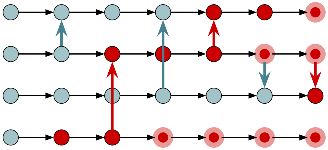
- interaction (temporal) network $G = (V, E)$

  n nodes V; m directed interactions $E = \{(u, v, t)\}$

  convenient to consider timestamped nodes $V = \{(u_i, t_i)\}$



model



- infection (activity)
  - infection starts externally
  - it may propagate only via interactions
  - infected nodes remain infected
  - no assumption about the model
- reports
  - reported infections $\mathcal{R} = \{(u, t)\}$
  - report can be later than activation
  - not all infected nodes are reported

problem definition

EPIDEMICRECOSTRUCTION

- input : given

  interactions $E = \{(u, v, t)\}$

  set of reported infections $\mathcal{R} = \{(u, t)\}$

  set of candidate seeds $C \subseteq V$

  integer k

- find : set of temporal paths P such that

  set of paths P spans $\mathcal{R}$

  seeds in P are in C

  number of seeds in P is at most k

  $\text{cost}(P \mid \mathcal{R}) = \sum_{e \in P} w(e)$ minimized

problem definition

EpidemicRecostruction

- input : given

  interactions $E = \{(u, v, t)\}$

  set of reported infections $\mathcal{R} = \{(u, t)\}$

  set of candidate seeds $C \subseteq V$

  integer k

- find : set of temporal paths P such that

  set of paths P spans $\mathcal{R}$

  seeds in P are in C

  number of seeds in P is at most k

  $\text{cost}(P \mid \mathcal{R}) = \sum_{e \in P} w(e)$ minimized

EpidemicRecostruction is **NP**-hard

related problem

MinDirSteinerTree

- input : given
  directed graph $H = (U, F, w)$ with edge weights w
  root node $r \in U$
  set of terminal nodes $R \subseteq U$
- find : directed tree T rooted at r such that
  T contais paths from r to all nodes in R
  $\sum_{e \in T} w(e)$ is minimized

related problem

MinDirSteinerTree

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  directed graph $H = (U, F, w)$ with edge weights w

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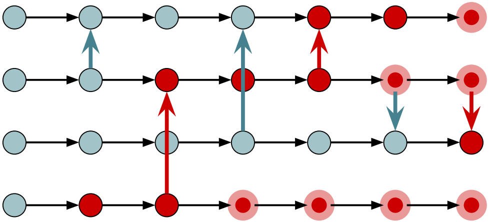
EpidemicRecostruction can be mapped to MinDirSteinerTree

transformation



add a dummy node, and

connect it with the earliest occurrence of each candidate seed,

with zero cost

solution idea

input

interactions E, reports $\mathcal{R}$, candidates C, integer k

transformation

1. construct a static graph $H = (U, F, w)$, where
   $U = V \cup \{d\}$ time-stamped nodes and dummy node d
2. edges from d to earliest occurrence candidate seeds
   set weight to α

solve MinDirSteinerTree on H

– subtrees of d are temporal paths P

– number of subtrees monotonic on weight α

– binary search on α, until less than k subtrees

solving MinDirSteinerTree

- MinDirSteinerTree is **NP**-hard
- recursive algorithm [Charikar et al., 1999]
- defined for recursion depth $i > 1$
- approximation guarantee $i(i-1)|X|^{\frac{1}{i}}$
- running time $\mathcal{O}(|V|^i|X|^i)$ [Huang et al., 2015]

  we use $i = 2$

main result

speedup

- MinDirSteinerTree pre-computes transitive closure of H
  - running time $\mathcal{O}(m^2)$
- need to calculate shortest paths for *'only'* $\mathcal{O}(n^2)$ pairs
  - a scan on E requiring $\mathcal{O}(nm)$ time [Huang et al., 2015]

proposition

for the EpidemicRecostruction problem, we can obtain approximation $2|n|^{\frac{1}{2}}$ in time $\mathcal{O}(mn)$

experimental evaluation

- datasets : synthetic, facebook, tumblr, students, and enron
- weights : $w(u, v, t) = \frac{1}{2}(|t - t_R(u)| + |t - t_R(v)|)$
- setting : simulate epidemic cascades with different models

  sample infections reports

  compare with ground truth
- baseline : one-hop extension
- evaluation metric : Matthews correlation coefficient

$$\mathrm{MCC} = \frac{\mathrm{TP} \cdot \mathrm{TN} - \mathrm{FP} \cdot \mathrm{FN}}{\sqrt{(\mathrm{TP} + \mathrm{FP})(\mathrm{TP} + \mathrm{FN})(\mathrm{TN} + \mathrm{FP})(\mathrm{TN} + \mathrm{FN})}}$$

experimental evaluation — results

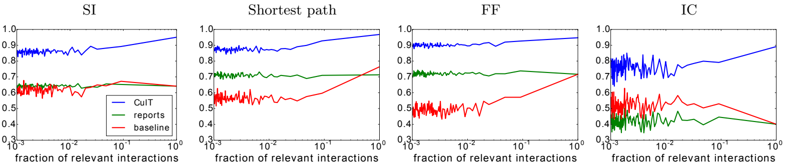


Figure 4: Effect of the fraction of interactions in the interaction history E that are relevant to the propagation. Reconstruction quality measured by MCC on the *Facebook* dataset, for different infection models.

reconstructing activity timelines in temporal networks

P. Rozenshtein, A. Gionis, N. Tatti, ECML PKDD 2017

the timeline reconstruction problem

- consider a set of entities
- entities can become *active* or *inactive*
- entities interact over time, forming a temporal network
- each interaction is attributed to an active entity
- can we reconstruct the activity timeline that explains best the observed temporal network?

the timeline reconstruction problem

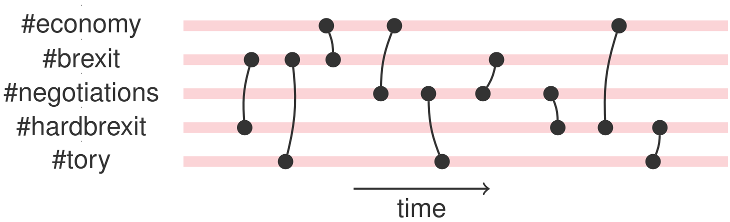
- consider a set of entities
- entities can become *active* or *inactive*
- entities interact over time, forming a temporal network
- each interaction is attributed to an active entity
- can we reconstruct the activity timeline that explains best the observed temporal network?
- assumption: being active is more costly, thus we want to minimize total activity time

the timeline reconstruction problem

- motivating example
- analyze a discussion in twitter about a topic (e.g., brexit)
- entities are hashtags
- two hashtags interact if they appear in the same tweet
- summarize the discussion by reconstructing a timeline
- pick a set of important hashtags and the time intervals they are active

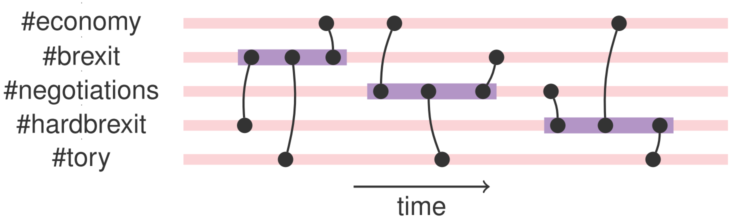
the timeline reconstruction problem

motivating example



the timeline reconstruction problem

motivating example



problem formalization

- given a temporal network $G = (V, E)$ with $E = \{(u, v, t)\}$
- find a set of intervals associated with nodes
  (the intervals that nodes are active)
  at most k per node
- that cover all edges, and
  - k-SUM-SPAN : minimize the sum of interval lengths
  - k-MAX-SPAN : minimize the max interval length

results

1-MAX-SPAN : solvable in linear time (related to 2-SAT)

1-SUM-SPAN : **NP**-hard

k-MAX-SPAN, $k > 1$: inapproximable

k-SUM-SPAN, $k > 1$: inapproximable

efficient and practical algorithms for hard problems

timeline reconstruction — case study

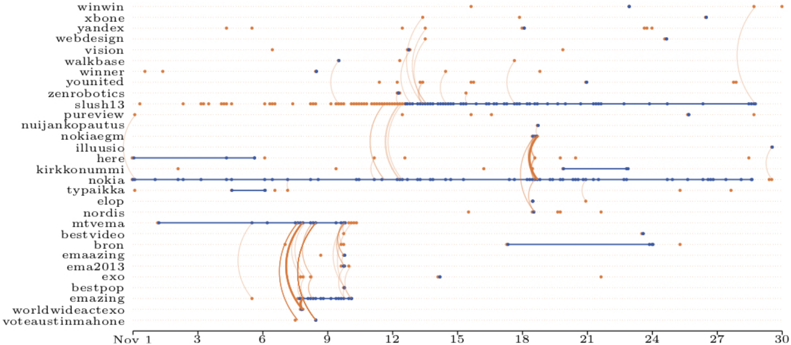


Fig. 7. Part of the output of `Inner` algorithm on Twitter dataset for November'13. Tags, co-occurring with hashtags `#slush13`, `#mtvema` and `#nokiaemg`. Activity intervals and active moments of interactions (hashtags' co-occurrences) are colored blue, inactive moments of interactions are colored orange. Only edges between an active and inactive hashtags are shown.

timeline reconstruction — case study

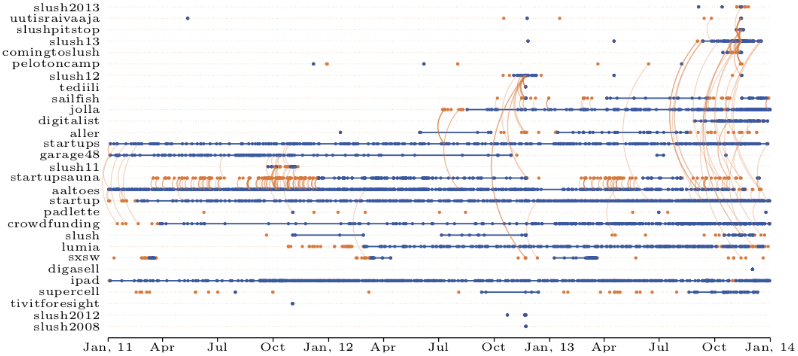


Fig. 8. Part of the output of `k-Inner` algorithm on Twitter dataset years 2011–2013 with $k = 3$. Tags, co-occurring with hashtag `#slush`. Activity intervals and active moments of interactions (hashtags' co-occurrences) are colored blue, inactive moments of interactions are colored orange. Only edges between an active and inactive hashtags are shown.

summary

- examples of mining temporal networks
  - temporal PageRank
  - maintaining sliding-window neighborhood profiles
  - reconstructing an epidemic over time
  - reconstructing activity timelines
- potential for new concepts, new problem definitions, new computational methods, and new applications

references

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