Cryptanalysis of the ESSENCE Hash Function

M. Naya-Plasencia[1] A. Röck[2] J-P. Aumasson[3]
Y. Laigle-Chapuy[1] G. Leurent[4] W. Meier[3] T. Peyrin[5]

[1]INRIA project-team SECRET, France
[2]Helsinki University of Technology (TKK), Finland
[3]FHNW, Windisch, Switzerland
[4]École Normale Supérieure, Paris, France
[5]Ingenico, France

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Outline

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Hash Functions

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Hash Functions

- **Symmetric cryptography**:
  Stream ciphers, Block ciphers, Hash functions
- Hash functions:
  - Given a message $\mathcal{M}$ of arbitrary length, a value $\mathcal{H}(\mathcal{M})$ of fixed length ℓ_h is returned
  - Many applications: MAC's (authentification), digital signatures...

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Security Requirements of Hash Functions

- Collision resistance:
  - Finding two messages $\mathcal{M}$ and $\mathcal{M}'$ so that $\mathcal{H}(\mathcal{M}) = \mathcal{H}(\mathcal{M}')$ must be "hard"
- Second preimage resistance:
  - Given a message $\mathcal{M}$ and $\mathcal{H}(\mathcal{M})$, finding another message $\mathcal{M}'$ so that $\mathcal{H}(\mathcal{M}) = \mathcal{H}(\mathcal{M}')$ must be "hard"
- Preimage resistance:
  - Given a hash $\mathcal{H}$, finding a message $\mathcal{M}$ so that $\mathcal{H}(\mathcal{M}) = \mathcal{H}$ must be "hard"
- Remark: We never say impossible

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Security Requirements of Hash Functions

A strict definition of "hard":

- Collision resistance
  - Generic attack needs $2^{\ell_h/2}$ hash function calls
    $\Rightarrow$ any attack requires at least as many hash function calls as the generic attack.
- Second preimage resistance and preimage resistance
  - Generic attack needs 2^{ℓ_h} hash function calls
    $\Rightarrow$ any attack requires at least as many hash function calls as the generic attack.

SHA-3 Competition [NIST]

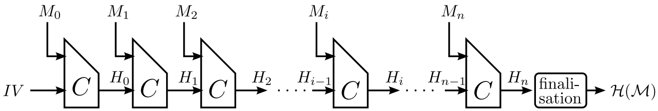
- Attacks against MD5, SHA-1,...
- Confidence in SHA-2 (standard) undermined
- Need of SHA-3: NIST has launched a public competition

SHA-3 Competition - Candidates

- 64 submissions (October 2008)
- 51 first round candidates
  - ESSENCE
- 14 second round candidates (July 2009)

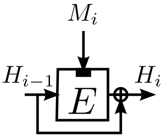
Merkle-Damgård

- Merkle-Damgård is an often used construction
  - Split message $\mathcal{M}$ into message blocks $M_0, M_1, \ldots, M_n$ of fixed size m
  - If M_n is not bit enough extend it to m bits: padding
  - H_i are the intermediate chaining values
  - If the one-way compression function C is collision resistant, then so is the hash function



Davies-Meyer

- Davies-Meyer is a method to construct a secure one-way function from a block cipher E
  - Secure under the "black-box" model (the block cipher has the required randomness properties and the attacker cannot use any special properties or internal details of E)



The ESSENCE Hash Functions

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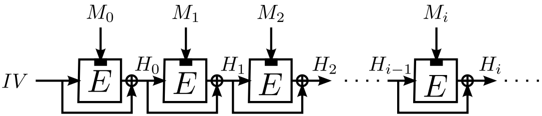
ESSENCE [Jason W. Martin]

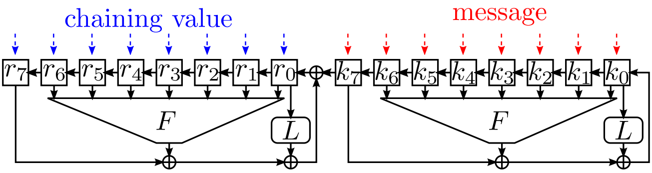
- First round candidate of the SHA-3 competition
- Bases on feedback shift registers
  - over 32-bit words for ESSENCE-256/224
  - over 64-bit words for ESSENCE-512/384
- Message block: 8 words
- Chaining value: 8 words

Structure

- Merkle-Damgård tree
  - A leaf hashes a fixed number of message blocks using MD
  - The inner nodes are combined again by MD
  - The height of the trees depends on a changeable parameter
  - The roots are combined with a final block containing the message length
- Davies-Meyer construction for the compression function



Block Cipher of the Compression Function



$32\times$ clocked

- *F*: bitwise non-linear function
- *L*: linear function on the whole word
- 32 reversible steps

Attack on ESSENCE

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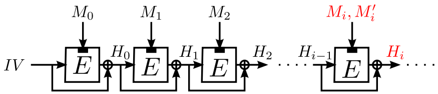
Principle

- Collision in compression function
- Using a differential path



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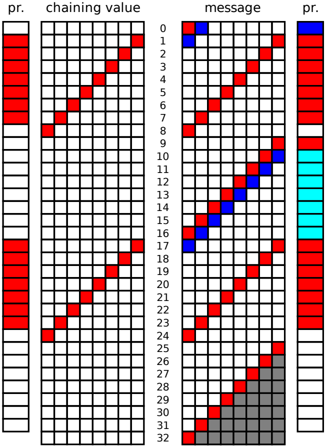
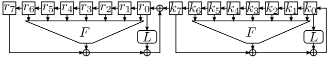
Differential Path - General

- For iterative structures
- Let Z_i, Z_i' denote the states after i (out of N) iterations starting from Z_0, Z_0'
  - Consider differences $\Delta_i = Z_i \oplus Z_i'$ for $0 \leq i \leq N$
  - Transition from Δ_i to Δ_{i+1} with certain probability
- Finally we want no difference in the chaining value

Differential Path





Differences:

- no difference
- α
- $\beta = L(\alpha)$
- unknown

Probabilities:

- $2^{-|\alpha|}$
- $2^{-|\beta|}$
- $2^{-|\alpha \vee \beta|}$
- 1

Condition:

$$\alpha \vee \beta \vee L(\beta) = \alpha \vee \beta$$

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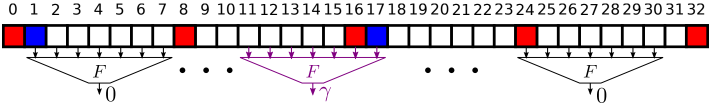
Exact Complexities

- Probabilities based on Hamming weight (HW) underestimates the real complexity of the attack:
  - e.g. a 1 bit difference has probability $2^{-8.4}$ to pass the 7 steps of F, and not 2^{-7} as we would guess from the HW
- For accurate estimates consider the whole path bitwise
  - Possible differences: $(\alpha_i, \beta_i, \gamma_i)$ with $0 \leq i \leq 32/64$ and $\beta = L(\alpha)$ and $\gamma = L(\beta)$
  - Have to test 2^{30} values for each each $(\alpha_i, \beta_i, \gamma_i)$



Probability of Complete Path - Bitwise

- Bitwise probability, independent of α

$(\alpha_i, \beta_i, \gamma_i)$	(0,0,0)	(0,0,1)	(0,1,0)	(0,1,1)
probability	1	0	$2^{-9.5}$	$2^{-9.1}$
$(\alpha_i, \beta_i, \gamma_i)$	(1,0,0)	(1,0,1)	(1,1,0)	(1,1,1)
probability	$2^{-24.4}$	0	2^{-23}	2^{-26}

- Gives two conditions for α:
  - $\neg\alpha \wedge \neg\beta \wedge \gamma = 0$
  - $\alpha \wedge \neg\beta \wedge \gamma = 0$

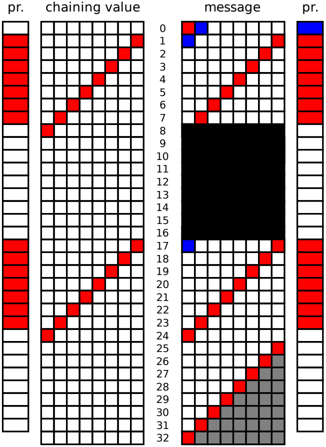
Complexity of Complete Path

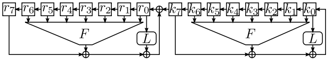
- Complexity for the α's used in our attack:

	differential path left	differential path right	generic method
ESSENCE-256	$2^{67.4}$	$2^{240.6}$	2^{128}
ESSENCE-512	$2^{134.7}$	$2^{478.9}$	2^{256}

- About $2^{15.4}$ pairs pass the whole path for ESSENCE-256 ($2^{37.1}$ for ESSENCE-512)

Idea: Computing the Middle Part





Differences:

- □ no difference
- α
- $\beta = L(\alpha)$
- unknown

Probabilities:

- $2^{-|\alpha|}$
- $2^{-|\beta|}$
- $2^{-|\alpha \vee \beta|}$
- 1

Conditions:

$$\neg\alpha \wedge \neg\beta \wedge \gamma = 0$$
$$\alpha \wedge \neg\beta \wedge \gamma = 0$$

Strategy of the Attack

- Compute many pairs that fulfill the middle part (step 8-17)
- Search among those one message pair that passes the rest of the path (step 0-8 and step 17-32)
- Try different chaining values (random starting messages) with our message pair to find a collision

Computing the Middle Part

8	$x_0 \oplus \alpha$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
9	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$
10	x_2	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$
11	x_3	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}
12	x_4	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}
13	x_5	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}
14	x_6	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}
15	x_7	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
16	$x_8 \oplus \alpha$	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
17	$x_9 \oplus \beta$	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$x_{16} \oplus \alpha$

- Let ℓ be the word size (32 or 64), $\beta = L(\alpha)$, $\gamma = L(\beta)$, $s = |\alpha \vee \beta|$ and $\mathcal{S} = \{i : \alpha_i \vee \beta_i = 1\}$

Computing the Middle Part - Bit Level

- For all bit-difference $(\alpha_i, \beta_i, \gamma_i)$, $0 \leq i < 32/64$:
  - Store bit-tuples $(x_1, \ldots, x_{15})_i$ passing F in the middle part:

    $e.g.: F(x_2, x_3, x_4, x_5, x_6, x_7, x_8\ )_i = F(x_2, x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha)_i$
  - Better: Store only those tuples which have a possibility to pass the rest of the path
- Number of tuples depending having the bit-differences:

$(\alpha_i, \beta_i, \gamma_i)$	$(0,0,1)$	$(0,1,0)$	$(0,1,1)$	$(1,0,0)$	$(1,0,1)$	$(1,1,0)$	$(1,1,1)$
	0	96	128	96	120	96	176
better	0	96	128	2	0	4	2

- Number of possibilities to choose $(x_1, \ldots, x_{15})_i$, $i \in \mathcal{S}$:

$$N_\alpha = 2^{|\alpha \wedge \neg\beta \wedge \neg\gamma|} \times 4^{|\alpha \wedge \beta \wedge \neg\gamma|} \times 96^{|\neg\alpha \wedge \beta \wedge \neg\gamma|} \times 2^{|\alpha \wedge \beta \wedge \gamma|} \times 128^{|\neg\alpha \wedge \beta \wedge \gamma|}$$

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Computing the Middle Part - Fix s Bits

$$L(\overbrace{x_7}^{s \text{ bits fixed}}) = x_0 \oplus \overbrace{x_8 \oplus F(x_1, x_2, x_3, x_4, x_5, x_6, x_7)}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_8}^{s \text{ bits fixed}}) = \overbrace{x_1 \oplus x_9 \oplus F(x_2, x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha)}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_9}^{s \text{ bits fixed}}) = \overbrace{x_2 \oplus x_{10} \oplus F(x_3, x_4, x_5, x_6, x_7, x_8 \oplus \alpha, x_9 \oplus \beta) \oplus \gamma}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_{10}}^{s \text{ bits fixed}}) = \overbrace{x_3 \oplus x_{11} \oplus F(x_4, x_5, x_6, x_7, x_8 \oplus \alpha, x_9 \oplus \beta, x_{10})}^{s \text{ bits fixed}}$$

$$\ldots$$

$$L(\overbrace{x_{14}}^{s \text{ bits fixed}}) = \overbrace{x_7 \oplus x_{15} \oplus F(x_8 \oplus \alpha, x_9 \oplus \beta, x_{10}, x_{11}, x_{12}, x_{13}, x_{14})}^{s \text{ bits fixed}}$$

$$L(\overbrace{x_{15}}^{s \text{ bits fixed}}) = x_{16} \oplus \overbrace{x_8 \oplus F(x_9 \oplus \beta, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15})}^{s \text{ bits fixed}}$$

Computing the Middle Part - Linear Systems

- We have 7 linear systems depending on α, $8 \leq j \leq 14$

$$L(x_j) = R_j$$

- x_j and R_j have together
  - 2ℓ bits (ℓ is the word length)
  - $2s$ bit fixed
- L gives ℓ equations
- Probability of a solution $2^{-(2s-\ell)}$ if the system has full rank

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Computing the Middle Part - Solving the Systems

- The position of the fixed bits is given by $\mathcal{S}$
- Using Gauss elimination we find $2s - \ell$ equation which must be satisfied to have a solution
- Order the $7(2s - \ell)$ equations depending on the variables they contain, so that changing the variables in the later equations has no influence on the results of the first ones

Computing the Middle Part - Finishing

- After solving the linear systems we have
  - In x_j,R_j all bit fixed, $8 \leq j \leq 14$
  - In $x_1, \ldots, x_7, x_{15}$ we have s bit fixed
  - In x_0, x_{16} no bit fixed
- Selecting the $\ell - s$ free bits of x_7 allows us to determine all the other free bits
  $\Rightarrow$ For each solution of the linear systems we have $2^{\ell - s}$ solution for the middle part for free
- In average, we find a solution for $x_0, \ldots, x_{16}$ in less than one call to the compression function

Final Complexity

- To find the optimal α
  - ESSENCE-256: Test all possible α
  - ESSENCE-512: Test all α's with HW ≤ 8 (limitation on the left side)

	differential path		generic method
	left	right	
ESSENCE-256	$2^{67.4}$	$2^{62.2}$	2^{128}
ESSENCE-512	$2^{134.7}$	$2^{116.1}$	2^{256}

Semi-Free-Start Collision on 29 rounds

	Initial values for *r*									Initial values for *k*								
	B0741769	BA2BA1A1	349A4DC8	54204D82	292006B1	80096194	D23020E1	9098A7EA		4CD35806	4759FB6D	3ED267E5	17641536	BE1F35ED	688B0C3C	DF126549	5FAE0827	
round	differences								round	differences								round
0	0	0	0	0	0	0	0	0	0	80102040	537874EB	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	80102040	1	537874EB	0	0	0	0	0	0	80102040	1
2	0	0	0	0	0	0	80102040	0	2	0	0	0	0	0	0	80102040	0	2
3	0	0	0	0	0	80102040	0	0	3	0	0	0	0	0	80102040	0	0	3
4	0	0	0	0	80102040	0	0	0	4	0	0	0	0	80102040	0	0	0	4
5	0	0	0	80102040	0	0	0	0	5	0	0	0	80102040	0	0	0	0	5
6	0	0	80102040	0	0	0	0	0	6	0	0	80102040	0	0	0	0	0	6
7	0	80102040	0	0	0	0	0	0	7	0	80102040	0	0	0	0	0	0	7
8	80102040	0	0	0	0	0	0	0	8	80102040	0	0	0	0	0	0	0	8
9	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0	80102040	9
10	0	0	0	0	0	0	0	0	10	0	0	0	0	0	0	80102040	537874EB	10
11	0	0	0	0	0	0	0	0	11	0	0	0	0	0	80102040	537874EB	0	11
12	0	0	0	0	0	0	0	0	12	0	0	0	0	80102040	537874EB	0	0	12
13	0	0	0	0	0	0	0	0	13	0	0	0	80102040	537874EB	0	0	0	13
14	0	0	0	0	0	0	0	0	14	0	0	80102040	537874EB	0	0	0	0	14
15	0	0	0	0	0	0	0	0	15	0	80102040	537874EB	0	0	0	0	0	15
16	0	0	0	0	0	0	0	0	16	80102040	537874EB	0	0	0	0	0	0	16
17	0	0	0	0	0	0	0	80102040	17	537874EB	0	0	0	0	0	0	80102040	17
18	0	0	0	0	0	0	80102040	0	18	0	0	0	0	0	0	80102040	0	18
19	0	0	0	0	0	80102040	0	0	19	0	0	0	0	0	80102040	0	0	19
20	0	0	0	0	80102040	0	0	0	20	0	0	0	0	80102040	0	0	0	20
21	0	0	0	80102040	0	0	0	0	21	0	0	0	80102040	0	0	0	0	21
22	0	0	80102040	0	0	0	0	0	22	0	0	80102040	0	0	0	0	80000040	22
23	0	80102040	0	0	0	0	0	0	23	0	80102040	0	0	0	0	80000040	38C32419	23
24	80102040	0	0	0	0	0	0	0	24	80102040	0	0	0	0	80000040	38C32419	3B50EAEF	24
25	0	0	0	0	0	0	0	0	25	0	0	0	0	80000040	38C32419	3B50EAEF	E9F738F8	25
26	0	0	0	0	0	0	0	0	26	0	0	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	26
27	0	0	0	0	0	0	0	0	27	0	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	27
28	0	0	0	0	0	0	0	0	28	0	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	28
29	0	0	0	0	0	0	0	0	29	80000040	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	29
30	0	0	0	0	0	0	0	80000040	30	38C32419	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	30
31	0	0	0	0	0	0	80000040	102040	31	3B50EAEF	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	21E1C70	31
32	0	0	0	0	0	80000040	102040	3336DACE	32	E9F738F8	D59E6BC4	519ECD90	8199374F	1B9B997C	A7EF91F9	21E1C70	1B715D5F	32

Conclusion

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Conclusion

- **Complexity:**
  - ESSENCE-*256*: $2^{67.4}$
  - ESSENCE-*512*: $2^{134.7}$
- **Why does the attack work?**
  - Message precessing is independent of chaining value
  - Precompute low probability part
  - Efficient solving of linear system
  - Exact probability by considering the bit path
  - Reduced cost by considering the whole path

Conclusion

- **Complexity:**
  - ESSENCE-*256*: $2^{67.4}$
  - ESSENCE-*512*: $2^{134.7}$
- **Why does the attack work?**
  - Message precessing is independent of chaining value
  - Precompute low probability part
  - Efficient solving of linear system
  - Exact probability by considering the bit path
  - Reduced cost by considering the whole path

HELSINKI UNIVERSITY OF TECHNOLOGY
Department of Information and Computer Science