Joint work with...

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... and many others
Algorithm synthesis

• Computer science: what can be automated?

• Can we *automate our own work*?

• Can we outsource algorithm design to computers?
  • *input*: problem specification
  • *output*: asymptotically optimal algorithm
Verification and synthesis

• Verification:
  • given problem $P$ and algorithm $A$
  • does $A$ solve $P$?

• Synthesis:
  • given problem $P$
  • find an algorithm $A$ that solves $P$?
Verification and synthesis

• Algorithm **verification** often difficult
  • easy to run into e.g. halting problem

• Algorithm **synthesis** is entirely hopeless?

• Not necessarily!
  • verifying **arbitrary** algorithms in model $M$
  • synthesising only “**nice**” algorithms in model $M$
Setting

• Our focus: distributed algorithms
  • multiple nodes working in parallel
  • complicated interactions between nodes
  • possibly also faulty nodes, adversarial behaviour

• Computational techniques in algorithm design can outperform human beings
Setting

• We do theory, not practice

• Desired outputs:
  • *algorithm design & analysis*
  • *lower-bound proofs*

• We want *provably correct algorithms*, not something that “seems to work”
Success stories (1/4)

• **Fault-tolerant digital clock synchronisation**
  
  • nodes have to count clock pulses modulo $c$
  
  • *self-stabilising algorithms*: reaches correct behaviour even if the starting state is arbitrary
  
  • *Byzantine fault tolerance*: some nodes may be adversarial
4 nodes
1 faulty node
3 states per node
always stabilises in at most 7 steps
Success stories (2/4)

• **Theorem:** any triangle-free $d$-regular graph has a cut of size $\left(\frac{1}{2} + \frac{0.281}{\sqrt{d}}\right)m$

• prior bound: $\left(\frac{1}{2} + \frac{0.177}{\sqrt{d}}\right)m$ (Shearer 1992)

• **Proof:** we design a *simple randomised distributed algorithm* that finds such cuts (in expectation)
Pick a random cut, change sides if at least \(\left\lfloor \frac{d+\sqrt{d}}{2} \right\rfloor\) neighbours on the same side
Success stories (3/4)

• Classical symmetry-breaking primitive:
  • input: directed path coloured with $n$ colours
  • output: directed path coloured with 3 colours

• Prior work: $\frac{1}{2} \log^*(n) \pm O(1)$ rounds

• New result: exactly $\frac{1}{2} \log^*(n)$ rounds for infinitely many $n$
Success stories (4/4)

• Any **locally checkable labelling problem**
  • maximal independent set, colouring …

• Setting: cycles, 2-dimensional grids, …

• Complexity is $O(1)$, $\Theta(\log^* n)$, or $\Theta(n)$

• **Synthesis possible for class $\Theta(\log^* n)$**
Key challenges

• A combinatorial search problem
  • find an object $A$ that satisfies these constraints…

• How to make the problem finite?
  • so that the problem is solvable at least in principle

• How to solve it in practice?
  • how to avoid combinatorial explosion
Key challenges

• Much easier to make the problem finite if we **fix some parameters**:
  • algorithm for \( n = 10 \) nodes?
  • algorithm for any \( n \), but maximum degree \( \Delta = 10 \)?

• How to **generalise**?
How to generalise

1. **Computer-inspired algorithms**
   - computer solves *small cases*, generalise the idea

2. **Generalise by induction**
   - computer solves the *base case*, prove inductive step

3. **Direct synthesis for the general case**
   - sit down and relax
How to generalise

1. Computer-inspired algorithms
   • example: *large cuts*

2. Generalise by induction
   • example: *clock synchronisation*

3. Direct synthesis for the general case
   • example: $O(\log^* n)$-time algorithms
LCLs on cycles

• Computer network = directed $n$-cycle
  • nodes labelled with $O(\log n)$-bit identifiers
  • each round: each node exchanges (arbitrarily large) messages with its neighbours and updates its state
  • each node has to output its own part of the solution
  • $time = number of rounds$ until all nodes stop
  • equivalently: $time = distance$ (how far to look)
LCLs on cycles

• LCL problems:
  • solution is globally good if it looks good in all local neighbourhoods
  • examples: vertex colouring, edge colouring, maximal independent set, maximal matching…
  • cf. class NP: solution easy to verify, not necessarily easy to find
LCLs on cycles

• **2-colouring**: inherently global
  • $\Theta(n)$ rounds

• **3-colouring**: local
  • $\Theta(\log^* n)$ rounds
LCLs on cycles

- Given an algorithm, it may be very difficult to verify
  - easy to encode e.g. halting problem
  - running time can be any function of $n$

- However, given an LCL problem, it is very easy to synthesise optimal algorithms!
LCLs on cycles

• LCL problem $\approx$ set of feasible local neighbourhoods in the solution

• Can be encoded as a graph:
  • node = neighbourhood
  • edge = “compatible” neighbourhoods
  • walk $\approx$ sliding window
LCLs on cycles

Neighbourhood $v$ is “flexible” if for all sufficiently large $k$ there is a walk $v \rightarrow v$ of length $k$

- equivalent: there are walks of coprime lengths
- “12” is flexible here, $k \geq 2$
LCLs on cycles

- **independent set**
  - Self-loops: $O(1)$

- **maximal independent set**
  - Flexible states: $\Theta(\log^* n)$

- **3-colouring**
  - Otherwise: $\Theta(n)$

- **2-colouring**
  - 12 → 21
LCLs on cycles

• Verification hard but synthesis easy:
  • construct graph, analyse its structure

• “Compactification”:
  • any LCL problem can be represented *concisely* as a graph
  • seemingly open-ended problem of finding an efficient algorithm is reduced to a simple graph problem
Beyond cycles

• Classification **undecidable** on 2D grids
  • “is this problem solvable in $O(\log^* n)$”

• But **1 bit of advice** is enough!
  • just tell me whether it is solvable in time $O(\log^* n)$
  • then I can find an optimal algorithm — at least in principle, but often also in practice
  • key insight: “**normal form**” for any such algorithm
$O(\log^* n)$
Future

• How far can we push these techniques?
  • immediate next steps: distributed algorithms in much more general graph families

• More focus on *meta-algorithmics*?
  • how to design algorithms for designing algorithms

• Algorithms for *lower bounds*?