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MAT282 Introduction to Discrete Mathematics

Midterm Exam 2, Wed 29 November, 8–11 a.m.

1. Give examples of graphs that contain:
   (a) neither an Eulerian tour (circuit) nor a Hamiltonian cycle;
   (b) an Eulerian tour but no Hamiltonian cycle;
   (c) a Hamiltonian cycle but no Eulerian tour;
   (d) both an Eulerian tour and a Hamiltonian cycle.

2. Given a graph $G = (V, E)$, define the *average degree* of its vertices as

$$\bar{\delta}_G = \frac{1}{|V|} \sum_{u \in V} \deg(u).$$

   Prove that for all planar graphs G it is the case that $\bar{\delta}_G < 6$, and that for trees T the average degree approaches the limit value $\lim \bar{\delta}_T = 2$, as the number of vertices grows.

3. A *hypercube graph* H_n has as vertices all the n-bit binary sequences $u \in \{0,1\}^n$, and vertices u and v are connected by an edge if and only if the corresponding binary sequences differ in exactly one position. Determine the chromatic number $\chi(H_n)$ for each n. (Justify your answer.)

4. Design some six-bit, single-error correcting linear code with 8 codewords. Write down the parity-check matrix and the set of codewords of your code. Show how you would use the given parity-check matrix to correct some word containing a single-bit error.

Grading: Problems 1 and 2 each 8 points, Problems 3 and 4 each 7 points, total 30 points.