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MAT282 Introduction to Discrete Mathematics

Final Exam, Wed 28 March 2001, 8–12 a.m.

1. A mid-size university department has six working groups developing the department's Teaching Quality Initiatives. Five professors (A,...,E) participate in these groups, according to the following scheme:

1: A, B, C	4: B, C, D
2: A, C, D	5: B, E
3: A, E	6: C, D, E

   Each working group meeting requires a full day, and obviously no two groups with common members can be scheduled for the same day. Design a graph-theoretic model of the situation, and determine on this basis how many days are needed for each group to hold at least one meeting.

2. The student association of the abovementioned department selects ten representatives to work on the Teaching Quality Initiatives, so that each of these ten persons participates in the workings of exactly one of the groups 1–6. Compute the number of ways the groups' student representations can be arranged, under the conditions that each group must have at least one student member, but at most as many as there are professor members in that group. (For the purposes of this problem, students are considered to be identical, so that it is only significant how *many* student representatives there are in each working group.)

3. Our example department has been awarded for its development work a Quality Diplom by the Rector, and each of the professors A,...,E may now hold the Diplom on their bookshelf one week at a time. Design for the Diplom a 10-week rotation schedule, during which each pair of professors who had worked together in some of the groups 1–6 exhanges the diplom exactly once. (The direction of the exchange, i.e. who forwards the Diplom to whom, does not matter.)

4. Find a closed form solution for the recurrence equation:

$$\begin{cases} a_0 = -1, \quad a_1 = 1, \\ a_n = 4a_{n-1} - 3a_{n-2} + 2^n, \quad n \geq 2. \end{cases}$$

5. A round pizza is cut into pieces by n knife cuts, so that any two cuts meet pairwise, but no three at the same point (cf. Figure 1). Prove that the cuts divide the pizza into $\binom{n}{0} + \binom{n}{1} + \binom{n}{2}$ pieces. (*Hint:* One way to solve the problem is to view the pizza as a plane graph.)

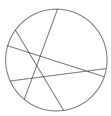

Figure 1: A sliced pizza

Grading: Each problem 12 points, total 60 points.