



**Thesis Title:** Adding Machines and  $P$ -adic Integers

## ABSTRACT

Adding machines are minimal homeomorphisms from Cantor set, i. e., all orbits are dense. It is proved in this work that stable Cantor sets are topologically conjugate to an adding machine with prime basis, without loss of generality. A typical example is the binary adding machine, which is topologically conjugate to the Feigenbaum attractor [milnor1985]. This example suggests a connection to the theory of  $p$ -adic numbers ( $p = 2$  case).

Moreover, general adding machines motivate a topological and algebraically generalization of this numbers; in this work they are called  $P$ -adic integers, where  $P$  denotes a sequence of primes. Here, it is given a topological and algebraically characterization of this numbers; namely, it is shown that  $\mathbb{Z}_P$ , the  $P$ -adic integers ring, is a Cantor set.

Furthermore, it is proven that the Cantor set  $\mathbb{Z}_P$  acts like an adding machine which is topologically conjugate to a general adding machine with basis  $P$ . Based on this fact, it is given a proof of the classification theorem for adding machines with prime basis.

**Keywords:**  $p$ -adic numbers,  $P$ -adic integers, *Adding machines*, Liapunov Stability, Classification theorem for adding machines.

[milnor1985] J. Milnor, *On the concept of attractor*, Communications in Mathematical Physics, 99 (1985), pp. 177–195.