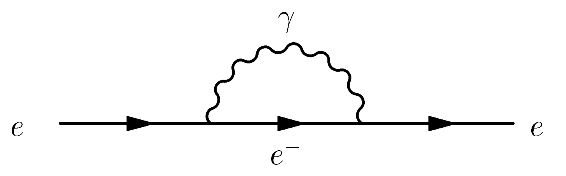
Theories of particle physics Fall 1998, Exercise 12

1. Write down the renormalized and bare Lagrangian of QED, as well as the counter-term Lagrangian. Calculate the renormalization constant Z_2 for the electron self-energy in QED in mass-shell, MS and $\overline{MS}$ schemes. (See e.g. Itzykson-Zuber p. 319–335, Bailin-Love, subsection 7.5 and chapter 11 (note that there might be some misprints around eq. (11.58)))
   Hint:



   Use dimensional regularization, Feynman gauge and results of problem 5 of exercise 11.

2. Calculate the β-function of QCD, and discuss its physical content. The β-function is defined as
$$\beta \equiv \lim_{2\omega\to 4^-} \frac{dg_R}{d\ln\mu},$$
   where the renormalized coupling constant is
$$g_R = g_B \mu^{\omega-2} Z_1^{-1} Z_2 Z_3^{\frac{1}{2}}$$
   when the renormalized fields are defined as
$$\psi_R = Z_2^{-\frac{1}{2}}\psi_B, A_R^\mu = Z_3^{-\frac{1}{2}} A_B^\mu.$$
   The Z's can be shown to be
$$\begin{aligned} Z_1 &= 1 - \tfrac{\alpha_C}{4\pi}\tfrac{16}{3}\tfrac{1}{2-\omega} + C_1, \\ Z_2 &= 1 - \tfrac{\alpha_C}{4\pi}\tfrac{8}{3}\tfrac{1}{2-\omega} + C_2, \\ Z_3 &= 1 + \tfrac{\alpha_C}{4\pi}\left(10 - \tfrac{4}{3}N_g\right)\tfrac{1}{2-\omega} + C_3, \end{aligned}$$
   where C are finite constants that are fixed by the renormalization scheme. 2ω denotes the number of space-time dimensions.

3. Calculate the color factors for the cross section of process $s\overline{s} \to \gamma g$.
   Hint: You can assume the fermions to be massless. You might want to notice that apart from the color factors the differential cross section is of the same form as in the pair annihilation process from exercise 9, problem 2.

Return the solved problems to the downstairs box in Laurikainen building on Monday December 7th at 15 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/

This is the last problem sheet. There are two (still...) preliminary choises for the exam date: December 17th and January 22nd. The first one has got more votes this far, the final date will be decided on the Wednesday lecture. I will put a note on the date to the web page.