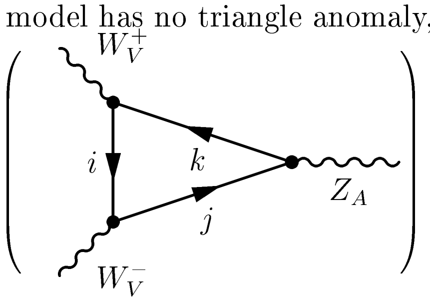
Theories of particle physics — Fall 1998, Exercise 11

1. Consider the decay of W boson. Show that in rest frame the decay width can be written as
$$\frac{d\Gamma}{d\Omega}(W \to f_1 f_2) = \frac{|T|^2}{64\pi^2}\frac{\sqrt{\lambda(m_W^2, m_1^2, m_2^2)}}{m_W^3}.$$
Calculate the decay width for the process $W^- \to e^-\overline{\nu}_e$. For unpolarized W^- the decay width can be written as
$$\frac{d\Gamma}{d\Omega}(W^- \to e^-\overline{\nu}_e) = \frac{1}{3}\sum_{\lambda=-1}^{1}\frac{d\Gamma}{d\Omega}(W_\lambda),$$
where the subscripts label the helicity of W. Show that the differential decay widths are
$$\frac{d\Gamma}{d\Omega}(W_0) = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta,$$
$$\frac{d\Gamma}{d\Omega}(W_{+1}) = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1-\cos\theta)^2,$$
$$\frac{d\Gamma}{d\Omega}(W_{-1}) = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1+\cos\theta)^2,$$
where θ is the angle between electron momentum and the polarization of W. The respective polarization vectors in the given frame of reference are
$$\varepsilon_0^\mu = (0,0,0,1), \varepsilon_{+1}^\mu = \frac{1}{\sqrt{2}}(0,-1,-i,0), \varepsilon_{-1}^\mu = \frac{1}{\sqrt{2}}(0,-1,i,0).$$
Use the above result to calculate the numerical value for the total decay width for W boson in SM. Use Particle Data Group book to get the latest numerical values (European Physical Journal C3 (1998) 1–794, can be found from the reference library (RT:Pa), or from the web http://pdg.lbl.gov/). The PDG book contains all experimental data on particle physics. Spend a silent moment browsing the book and contemplating the meaning of thousands of numerical values for all imaginable and unimaginable measurements and particle properties. (Historical note: The first version of Particle Data Book was written as a table by professor emeritus Matts Roos.)
Hint: Remember that there are three colours of each quarks. See section 4.4 of Halzen-Martin. You can ignore the masses of final-state fermions lighter than W.

2. Show that (9.2.30) is invariant in the Becchi-Rouet-Stora transformations (9.2.31). Write down the corresponding Ward identities.
Hint: See the lectures. Alternative derivation is given in Bailin-Love, appendix B.

3. Show that the standard model has no triangle anomaly, by showing that

$$\sum_{i,j,k=q,\ell}$$



$= 0,$

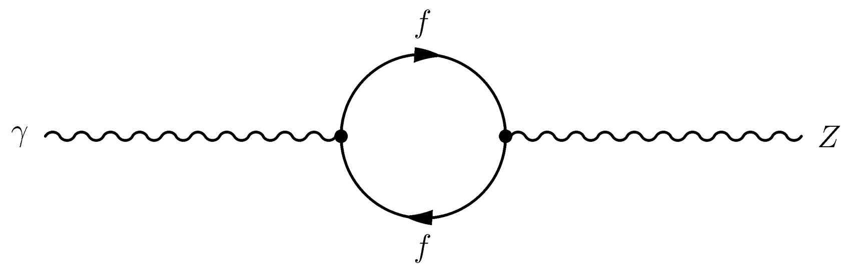
where $W_V^{\pm}$ and Z_A denote the vector and axial parts in the coupling of the respective gauge boson and the fermion, and i, j, k are all the quarks and leptons.
Hint: It is enough to show that

$$\sum_{i=q,\ell} g(W_V^+ ii) g(W_V^- ii) g(Z_A ii) = 0,$$

where g is the respective coupling constant. Note that the cancellation occurs separately for each family.

For further hints, see: e.g. Cheng and Li or Itzykson and Zuber, or section 15.5 of Bailin-Love (1993 rev. ed.). In-depth discussion of anomalies can be found from Weinberg: tQToF2, chapter 22, and especially section 22.4.

4. Consider the mixing of photon and Z-boson in SM in one-loop order, where the fermion loop is summed over all quarks and leptons:



Show that the mixing is zero when $\sin^2\theta_W = 3/8$. This number can also be obtained from grand unified theories a bit differently.
Hint: You do not have to calculate the integral, unless you desperately want to do it. The result is independent of number of families (why?), so it is enough to just show this for one family of quarks and leptons. Make up a convincing argument why the fermion masses can be ignored in the integral.

5. Derive the following integral that appears in the dimensional regularization method:

$$\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} (k^2 + 2p\cdot k - m^2 + i\epsilon)^{-n} = i(-1)^n \frac{\Gamma(n-\omega)}{\Gamma(n)} \frac{(m^2+p^2)^{\omega-n}}{(4\pi)^{\omega}},$$

where 2ω denotes the number of space-time dimensions. Calculate the above integral explicitly in the limit $\omega \to 2^-$ keeping all divergent and finite terms. Write down the result explicitly in case $n = 2$. (You should get something proportional to $1/(2-\omega) + \Gamma'(1) + \ln 4\pi$.) Differentiate the integral with respect to p_μ, ... to get

$$\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} A(k^2 + 2p\cdot k - m^2 + i\epsilon)^{-n}$$

where A denotes k_μ, $k_\mu k_\nu$ or k^2. Convince yourself that you can make all momentum integrals to be of this form by using Feynman trick (eq. (9.2.11) of Chaichian-Nelipa).
Hint: Bailin-Love, page 81 and around. You need only take into account cases when $n \geq 1$ is integer.

Return the solved problems to the downstairs box in Laurikainen building on Monday November 30th at 15 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/