1. Show that

$$(2\pi)^4\delta^4(p_3+p_4-p)\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

is Lorentz invariant. The cross section is generally given by

$$d\sigma = \frac{(2\pi)^4\delta^4(p_1+p_2-p_f)}{\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}}\,|\langle f|T|i\rangle|^2 \prod_f \frac{d^3\vec{p}_f}{(2\pi)^3 2E_j}.$$

Use this to derive the cross section for scattering $a_1 + a_2 \to b_1 + b_2$ in the center of mass frame:

$$\frac{d\sigma}{d\Omega} = \frac{|\langle f|T|i\rangle|^2}{64\pi^2 s}\left(\frac{|\vec{p}_f|}{|\vec{p}_i|}\right),$$

where $d\Omega$ is the element of solid angle about some outgoing particle (see e.g. Halzen-Martin exercise 4.2, or Bjorken-Drell, Cheng-Li app. A, Perle chapter 6). Express the initial and final momenta by using the Mandelstam variables s, t, u.
Hint: Show $d^3p/(2E) = \delta(p^2 - m^2)\Theta(p_0)d^4p$ (Renton p. 152). See Halzen-Martin section 4.3.

2. (solving this problem will give extra points) Draw the lowest-order Feynman diagrams for the pair annihilation process in QED

$$e^+(p_1,s_1) + e^-(p_2,s_2) \to \gamma(k_1,\varepsilon_1) + \gamma(k_2,\varepsilon_2).$$

Write down the transition amplitude using Feynman rules. Calculate the differential cross section of the process as a function of Mandelstam variables s, t, u (taking the electron mass into account). You can assume that the initial electrons are unpolarized. Verify that at high-energy limit the squared amplitude is proportional to

$$\frac{u}{t} + \frac{t}{u}.$$

What would the angular distribution of outgoing photons look like in high-energy e^+e^- collider, like LEP?
Hint: See the lectures, section 7.2.2, and exercise 6.22 of Halzen and Martin.

3. Derive the total cross section for the neutral current (Z exchange) scattering $\nu_\mu + e^- \to \nu_\mu + e^-$ in the laboratory frame of reference (=initial electron at rest).
Hint: Renton p. 277, Okun p. 130, 201, Cheng-Li p. 365. You may assume that $q^2 << M_Z^2$ and ignore mass of the electron in the *final* state (why?).

Return the solved problems to the downstairs box in Laurikainen building on Monday November 16th at 12 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/