1. The Majorana spinor operator is defined as

$$\chi(x) = \sum_{s=\pm\frac{1}{2}} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} \left[b(k,s)u(k,s)e^{-ik\cdot x} + \eta b^\dagger(k,s)v(k,s)e^{ik\cdot x} \right],$$

   where $|\eta| = 1$. Show that $\chi^c = \eta^* \chi$. Dirac spinor operator ψ can be defined as a sum of two Majorana spinor operators having opposite phase factor η:

$$\psi = \frac{1}{\sqrt{2}} \left(\chi_1 + \chi_2 \right).$$

   What are the creation and annihilation operators c and d of the Dirac spinor expressed in terms of Majorana operators $b_{1,2}$? Show that c and d obey canonical anticommutation relations.
   Hint: $v = u^c$ and $u = v^c$ ($\psi^c = C\overline{\psi}^T$).

2. Derive the commutation relations for the creation and annihilation operators of real scalar fields $a^\dagger(p)$, $a(p)$ from the assumption that the particle states are symmetric under change of particles. From this derive the commutation relation for the field operator $\hat{\psi}(x)$ and the canonical impulse $\hat{\Pi}(x)$.

3. Assume that you have a theory containing a heavy real scalar field ϕ and a light fermion field ψ having interaction Hamiltonian $\hat{H}_I = \lambda \hat{\overline{\psi}} \hat{\psi} \hat{\phi}$. Write down the transition amplitude for the decay of the boson (momentum p_0) to two fermions (momenta p_1, p_2) in leading order in terms of the spinor eigenvectors u and v, that is, calculate the matrix element

$$S_{fi} = -i \int d^4x \langle p_1 p_2 | \hat{H}_I | p_0 \rangle.$$

   Would it make any difference if the fermion field would be Dirac or Majorana field?

Return the solved problems to the downstairs box in Laurikainen building on Monday October 26th at 12 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/