1. Conjugation operator C is usually defined in group theory to be the operator that leaves $\psi^T C\psi$ invariant, where ψ is a representation of some group. If the group in question is homogenous Lorentz group and ψ are spin-$\frac{1}{2}$ representations of it, then C should satisfy $C\gamma^\mu C^{-1} = -\gamma^{\mu T}$, $C^\dagger C = 1$ (convince yourself that you can define such an object). *(a)* Replace the Dirac spinor ψ in the Lagrangian of QED (C&N section 2.3) with $\psi \to \psi^c = C\gamma^0\psi^*$ and show that everything remains the same — except the electric charge changes in sign. (*b*) Show that $\psi^T C\psi$ is really a Lorentz singlet (at least in infinitesimal transformations).
   Hint: For literature see e.g. Peskin and Schröder p. 64, Halzen and Martin, ...
   (This way you could create a *Majorana* mass term $m_M\psi^T C\psi$ for spin-$\frac{1}{2}$ particles carrying no unbroken $U(1)$ quantum numbers, like electric charge. Neutrinos would be an obvious candidate. Obviously(?) lepton number conservation would however be violated.)

2. Show that
$$\int_{-\infty}^{\infty}\left(\prod_i \frac{dy_i}{\sqrt{2\pi}}\right)\exp\left(-\frac{1}{2}y_i A^{ij} y_j\right) = (\det A)^{-\frac{1}{2}}.$$

3. What is
$$\int \prod_i d\eta_i^* \prod_i d\eta_i \exp\left(-\eta_i^* A^{ij}\eta_j\right)$$
   for anticommuting Grassman variables?

4. Study the $\lambda\phi^4$ model specified by the Lagrangian
$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4. \tag{1}$$
   where ϕ is a real scalar field. Beginning from this Lagrangian derive the path-integral form of the vacuum-vacuum transition amplitude following the lead of Chaichian and Nelipa eqs. (4.1.18–22). Write the equivalent of eq. (4.1.22) explicitly in discrete form (like in eqs. (4.1.14,17,...)) to familiarize yourself with the notation. Could you in principle calculate the amplitude (in discretized form) numerically?

Return the solved problems to the downstairs box in Laurikainen building on Monday October 19th at 12 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/