1. Consider the fields

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, R = e_R,$$

of the standard model. Write down the transformation of these fields under the electroweak symmetry group $SU(2)_L \times U(1)_Y$ explicitly. Then assume that field H obtains a vacuum expectation value, which can be chosen to be $\langle H \rangle = (0, v)^T$. Show that there is a residual symmetry $U(1)_Q$ that leaves the vacuum state $\langle H \rangle$ invariant. Find the generator of this symmetry group and write down explicitly the transformation properties of fields H, L and R under $U(1)_Q$ transformation. What are the $U(1)_Q$ quantum numbers, i.e. the electric charges, of the fields?
Hint: See Chaichian and Nelipa eqs. (6.1.5–6'), or e.g. Halzen and Martin, chapter 15.

2. Study in detail spontaneous symmetry breaking in locally $U(1)$ invariant theory having Lagrangian

$$\mathcal{L} = \overline{\psi} i\gamma^\mu D_\mu \psi + (D^\mu \phi)^* (D_\mu \phi) + m^2 \phi^* \phi - \frac{1}{4}\lambda(\phi^*\phi)^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu},$$

where $D_\mu = \partial_\mu + igA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and ϕ, ψ and A_μ are scalar, spinor and vector fields. What happens to the Goldstone boson? Can A_μ be a electromagnetic field? How could you make the fermion massive?
Hint: See the lecture notes. Also Bailin and Love, section 13.5, might prove useful.

3. Study the Lagrangian for the scalar and the gauge sector of the standard model

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) - V(H) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a,$$

where

$$V(H) = -\mu^2 H^\dagger H + \frac{1}{2}\lambda \left(H^\dagger H\right)^2.$$

Perform the spontaneous symmetry breaking. What happens to the Goldstone bosons? Diagonalize the gauge boson mass matrix and find the masses of the physical gauge bosons in terms of experimentally measurable quantities (e, G_F and θ_W). Show that $M_W/M_Z = \cos\theta_W$.
Hint: Halzen and Martin, sections 15.2–3, might be helpful.

4. Show that that the conjugate Higgs doublet $\tilde{H} = -i\tau_2 H^*$ transforms in (at least infinitesimal) $SU(2)$ transformations like Higgs doublet H. What are the electrical charges of the components of the conjugate doublet $\tilde{H}$? Show that the Yukawa terms

$$\mathcal{L}_{Yukawa} = -\lambda_d^{ij} \overline{Q_i} H d_{Rj} - \lambda_u^{ij} \overline{Q_i} \tilde{H} u_{Rj} - \lambda_e^{ij} \overline{L_i} H e_{Rj} + h.c.$$

are invariant under the electroweak group $SU(2)_L \times U(1)_Y$. Given same VEV for the Higgs field as in problem 1, what are the masses of the leptons and quarks expressed in terms of Yukawa couplings and Higgs VEV?
Hint: Halzen and Martin, chapter 15, could still be useful...

Return the solved problems to the downstairs box in Laurikainen building on Monday October 12th at 12 o'clock at latest. There are two problems solving sessions: Tuesdays 16-18 in the Exercise room (harjoitussali) in the basement of the Laurikainen bldg [English] and Fridays 10-12 in Hall 5, Tallqvist bldg [Finnish].
The www-page of this course is: http://www.physics.helsinki.fi/%7Epuolamak/hitu/