Linear State-Space Model with Time-Varying Dynamics

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Motivation

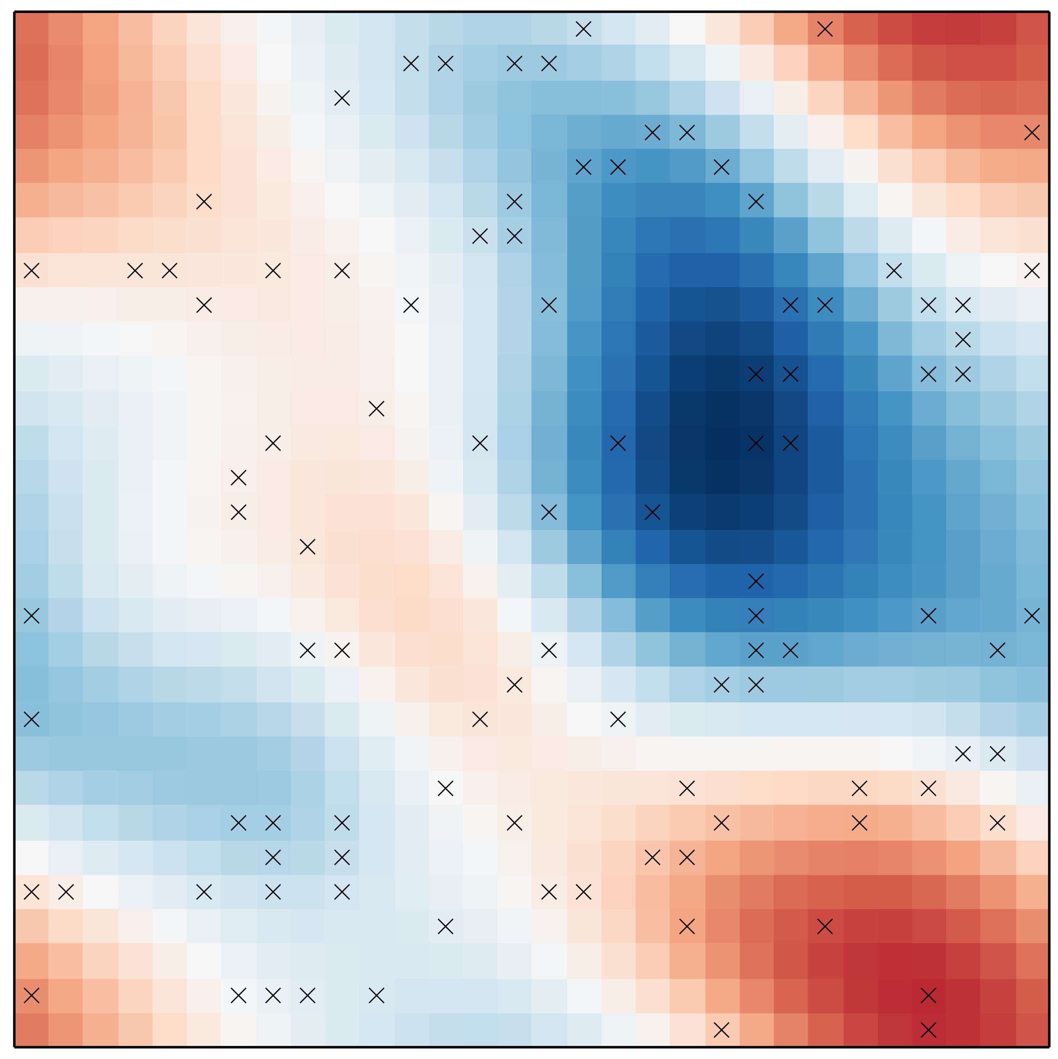
Statistical modelling of physical processes whose parameters vary in time.

Domain

- Spatio-temporal physical processes
- (Stochastic) linear partial differential equations (PDE)
- Arbitrary measurement locations
- Measurements at regular time intervals

Focus of the contribution

Parameters of the PDE vary in time (e.g., wind direction changes)



Snapshot of an advection-diffusion process.
Crosses denote measurement locations.

Model

Linear state-space model:

$$\text{Observations:} \quad \mathbf{y}_n = \mathbf{C}\mathbf{x}_n + \text{noise},$$
$$\text{Latent states:} \quad \mathbf{x}_n = \mathbf{W}_n\mathbf{x}_{n-1} + \text{noise},$$

Time-varying state dynamics matrix $\mathbf{W}_n$ as a time-varying linear combination of matrices $\mathbf{B}_k$:

$$\text{State dynamics matrix:} \quad \mathbf{W}_n = \sum_{k=1}^{K} s_{kn}\mathbf{B}_k,$$
$$\text{Mixing weights:} \quad \mathbf{s}_n = \mathbf{A}\mathbf{s}_{n-1} + \text{noise},$$

Inference

- Use the Bayesian framework.
- Approximate posterior using variational Bayes (VB).
- Factorize with respect to the variables:

$$p(\mathbf{X},\mathbf{C},\mathbf{B},\mathbf{S},\mathbf{A},\Theta|\mathbf{Y}) \approx q(\mathbf{X})q(\mathbf{C})q(\mathbf{B})q(\mathbf{S})q(\mathbf{A})q(\Theta).$$

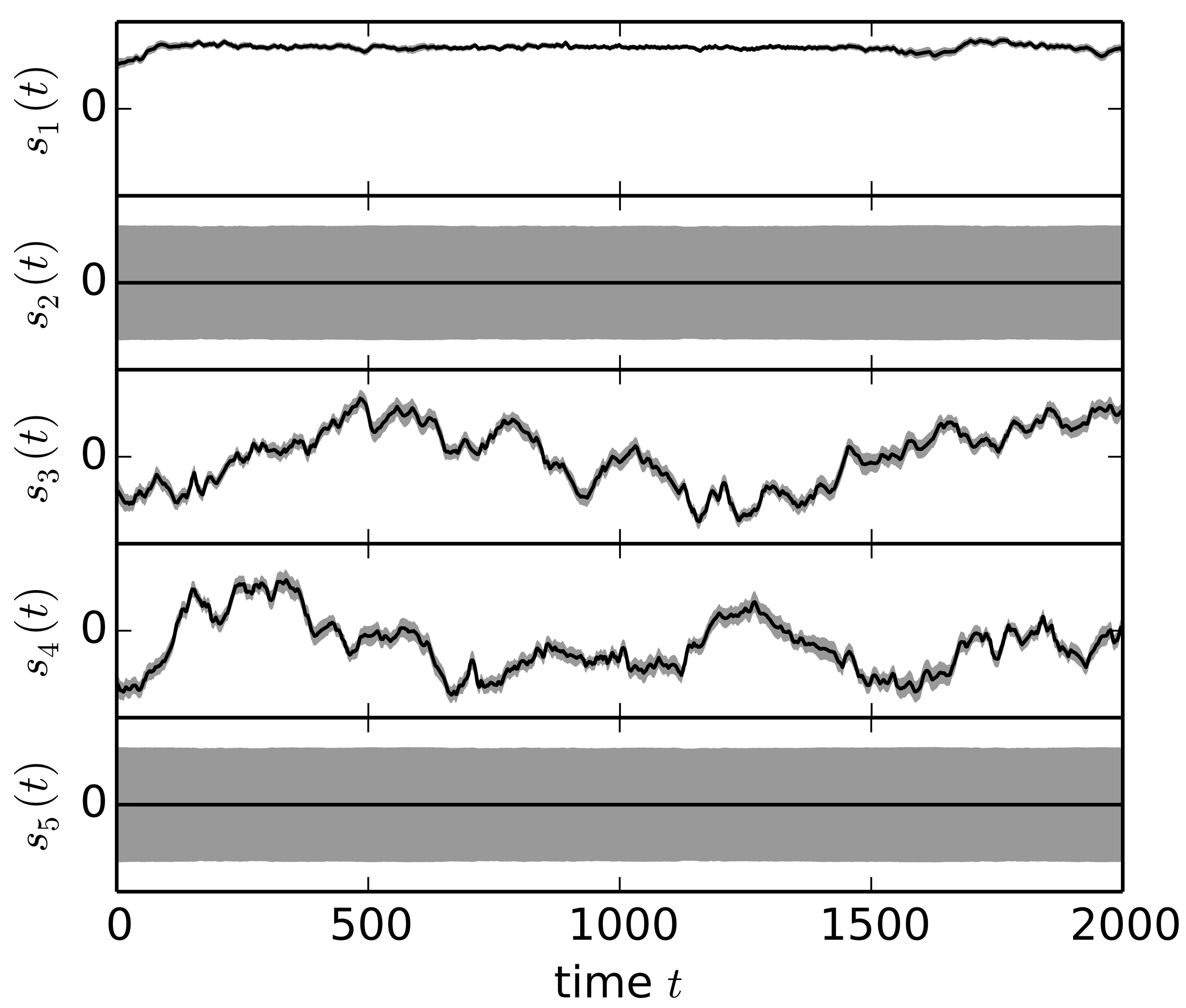
- Update each $q(\ldots)$ in turns (VB-EM algorithm).
- The method is available in BayesPy, a Python package for VB inference.

Stochastic Advection-Diffusion Process with Time-Varying Advection

$$\text{Simulated process:} \quad \frac{\partial f}{\partial t} = \delta\nabla^2 f - \mathbf{v}\cdot\nabla f + R,$$

- Velocity field $\mathbf{v}$ changes in time.
- Source R is stochastic.
- 100 measurement locations
- Five experiments: predictive RMSE for temporal gaps

Method	1	2	3	4	5
Standard LSSM	104	107	102	94	104
LSSM with switching dynamics	106	117	113	94	102
LSSM with time-varying dynamics	**73**	**81**	**75**	**67**	**82**



Posterior of the mixing weights $\mathbf{s}_k(n)$.

one constant component ~ average dynamics
two varying components ~ velocity field

The method was implemented as a part of a variational Bayesian Python package, BayesPy, available at GitHub and PyPI.
The scripts and data for reproducing the results are available at `http://users.ics.aalto.fi/jluttine/ecml2014`.