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Default Logic: From Theory to Applications

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Goals

- A framework for implementing nonmonotonic reasoning
- Implementation of a subclass of default logic (corresponding to normal logic programs) using the framework
- Application methodology for default reasoning

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Motivation

- Nonmonotonic logics provide compact representations (frame problem, qualification problem, rules with exceptions).
- Semantics, formal properties, relationships and complexity of nonmonotonic formalisms well-understood.
- Few practical applications (lack of efficient implementations)

☞ Breakthroughs in implementation techniques

Efficient implementations emerging

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Example. The **smodels** systems implementing a subclass of default logic [Niemelä and Simons, 1996]

Challenging blocks world planning problems:

Problem	Number of steps	Number of ground rules	Time (s)
large.c	8	81682	(first solution) 23.2
(15 blocks)	7	72528	(no solutions) 6.0
large.d	9	128000	(first solution) 47.1
(17 blocks)	8	115110	(no solutions) 11.5
large.e	10	191622	(first solution) 101
(19 blocks)	9	174100	(no solutions) 17.3

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Course Outline

- Introduction to default logic
- Complexity and expressivity
- Implementing default reasoning
- Logic programs with the stable model semantics
- Implementing the stable model semantics
- Rules as a constraint programming framework
- Planning

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Material

- Automating Default Reasoning
- Logic Programming with Stable Model Semantics as a Constraint programming Paradigm
- The **smodels** system: `http://www.tcs.hut.fi/smodels`

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Introduction to Default Logic

- Default logic = classical logic + default rules
- Default rules:
$$\frac{\varphi : \psi_1, \ldots, \psi_n}{\gamma}$$

φ: *prerequisite*
ψ_i: a *consistency condition* or *justification*
γ: *consequent*

'If φ is derivable and, for all i $(1 \leq i \leq n)$, $\neg\psi_i$ is not derivable, then derive γ".

Example. $\dfrac{quaker(X) : pacifist(X)}{pacifist(X)}$

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Examples.

- Rules with exceptions:
$\dfrac{bird(X) : \neg ab_f(X)}{fly(X)}$
$penguin(X) \rightarrow ab_f(X)$
$bird(X) \wedge broken\_wing(X) \rightarrow ab_f(X)$
$foot\_in\_concrete(X) \rightarrow ab_f(X)$
$\vdots$
- Frame axioms:
$\dfrac{on(X,Y,T) \wedge succ(T,T_1) : \neg moving(X,T)}{on(X,Y,T_1)}$
$move(X,Y,T) \rightarrow moving(X,T)$
$\vdots$

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- Knowledge is represented by a default theory: $(\mathcal{D}, W)$
  W: a set of first-order formulae
  $\mathcal{D}$: a set of defaults
- Default rule $\dfrac{\varphi : \psi_1, \ldots, \psi_n}{\gamma}$
  monotonic rule $\dfrac{\varphi}{\gamma}$ + consistency requirements $\{\psi_1, \ldots, \psi_n\}$
  ☞ Default reasoning = rule-based reasoning + consistency requirements
- **Rule-based reasoning**:
  classical logic + monotonic rules $\dfrac{\alpha_1, \ldots, \alpha_m}{\gamma}$
  Example. From $\{a\}$ and $\{\dfrac{\neg\neg a}{c}, \dfrac{\neg\neg c, a}{d}\}$
  d can be derived

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Rule-Based Reasoning

$W \vdash_R \varphi$ holds iff there is a finite sequence of formulae $\varphi_1, \ldots, \varphi_n (= \varphi)$ such that, for every $i \in \{1, \ldots, n\}$,

1. $W \cup \{\varphi_1, \ldots, \varphi_{i-1}\} \models \varphi_i$ or
2. there is a rule $\dfrac{\alpha_1, \ldots, \alpha_m}{\varphi_i} \in R$ such that $\{\alpha_1, \ldots, \alpha_m\} \subseteq \{\varphi_1, \ldots, \varphi_{i-1}\}$.

Example. $W \vdash_R d$ holds :

1.	$\neg\neg a$	By CL (from W)
2.	c	By $\dfrac{\neg\neg a}{c}$
3.	$\neg\neg c$	By CL
4.	a	By CL
5.	d	By $\dfrac{\neg\neg c, a}{d}$

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- Equivalently:
  $W \vdash_R \varphi$ holds iff there is a finite sequence of rules $r_1, \ldots, r_n$ in R such that
  1. $W \cup \{\mathrm{cons}(r_1), \ldots, \mathrm{cons}(r_{i-1})\} \models \mathrm{pre}(r_i)$, for $i \in \{1, \ldots, n\}$,
  2. $W \cup \{\mathrm{cons}(r_1), \ldots, \mathrm{cons}(r_n)\} \models \varphi$.
- Properties of the closure $\mathrm{Cn}_R(W) = \{\varphi \mid W \vdash_R \varphi\}$
  - $W \subseteq \mathrm{Cn}_R(W)$
  - closed under CL: $\mathrm{Cn}(\mathrm{Cn}_R(W)) \subseteq \mathrm{Cn}_R(W)$
  - closed under R:
    For all $r \in R$, $\mathrm{pre}(r) \subseteq \mathrm{Cn}_R(W)$ implies $\mathrm{cons}(r) \in \mathrm{Cn}_R(W)$
    $(\{\mathrm{cons}(r) \mid r \in R, \mathrm{pre}(r) \subseteq \mathrm{Cn}_R(W)\} \subseteq \mathrm{Cn}_R(W))$
  - $\mathrm{Cn}_R(W) = \mathrm{Cn}(W \cup \{\mathrm{cons}(r) \mid r \in R, \mathrm{pre}(r) \subseteq \mathrm{Cn}_R(W)\})$
- $\mathrm{Cn}_R(W)$ is the smallest set S such that
  $S = \mathrm{Cn}(W \cup \{\mathrm{cons}(r) \mid r \in R, \mathrm{pre}(r) \subseteq S\})$

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Semantics of Default Rules

- The set of conclusions not unique:

$$\frac{quaker(X) : pacifist(X)}{pacifist(X)} \quad (1)$$

$$\frac{republican(X) : \neg pacifist(X)}{\neg pacifist(X)} \quad (2)$$

$$quaker(nixon) \quad (3)$$

$$republican(nixon) \quad (4)$$

  From (3) and (1) follows $pacifist(nixon)$
  From (4) and (2) follows $\neg pacifist(nixon)$
- Intuitive reading circular!

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- A default theory can induce alternative acceptable sets of beliefs E (extensions) which should satisfy:
  (i) E contains W, (ii) E is deductively closed under CL
  (iii) E is closed under $\mathcal{D}$: all defaults that are "applicable" with respect to E have been applied,
  (iv) every formula in E justified by $(\mathcal{D}, W)$/grounded in $(\mathcal{D}, W)$, i.e. "derivable" from W and applicable defaults.
- Applicable defaults given E: $\frac{\varphi : \psi_1, ..., \psi_n}{\gamma}$ where for all $\psi_i, \neg\psi_i \notin E$
- Corresponding applicable rules given E
  $\mathcal{D}_E = \{\frac{\mathrm{pre}(d)}{\mathrm{cons}(d)} \mid d \in \mathcal{D}, \text{for all } \psi \in \mathrm{just}(d), \neg\psi \notin E\}$.
- If $\mathrm{Cn}_{\mathcal{D}_E}(W) \subseteq E$, then (i–iii) hold.
- If $E \subseteq \mathrm{Cn}_{\mathcal{D}_E}(W)$, then (iv) holds.

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Definition. Let $(\mathcal{D}, W)$ be a default theory. Then E is an extension of $(\mathcal{D}, W)$ iff $E = \mathrm{Cn}_{\mathcal{D}_E}(W)$.

Example. $\mathcal{D}$: $\frac{q(n) : p(n)}{p(n)}$, $\frac{r(n) : \neg p(n)}{\neg p(n)}$ $\quad W$: $q(n)$, $r(n)$

- $E_1 = \mathrm{Cn}(W \cup \{p(n)\})$ is an extension:
  As $\neg p(n) \notin E_1$, $\mathcal{D}_{E_1} = \{\frac{q(n)}{p(n)}\}$ and $\mathrm{Cn}_{\mathcal{D}_{E_1}}(W) = E_1$.
- So is $E_2 = \mathrm{Cn}(W \cup \{\neg p(n)\})$.
- But $E_3 = \mathrm{Cn}(W)$ is not:
  As $\neg p(n), \neg\neg p(n) \notin E_3$, $\mathcal{D}_{E_3} = \{\frac{q(n)}{p(n)}, \frac{r(n)}{\neg p(n)}\}$ and $\mathrm{Cn}_{\mathcal{D}_{E_3}}(W) = \mathrm{Cn}(W \cup \{p(n), \neg p(n)\}) \neq E_3$.

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- Existence of extensions not guaranteed:
  Assume that $(\mathcal{D}, W)$ has an extension E where $\mathcal{D} = \{\frac{: p}{\neg p}\}, W = \{\})$.
  If $\neg p \in E$ then $\mathcal{D}_E = \{\}$ and $E = \mathrm{Cn}_{\{\}}(\{\}) = \mathrm{Cn}(\{\})$ not containing $\neg p$.
  Hence, $\neg p \notin E$. Then $\mathcal{D}_E = \{\frac{}{\neg p}\}$ and $\neg p \in \mathrm{Cn}_{\mathcal{D}_E}(\{\})$, a contradiction.
- Extensions are minimal: if E is an extension, then there is no extension E' such that $E' \subset E$.
- The extensions of $(\mathcal{D}, W)$ are consistent provided that $\mathrm{Cn}_{\mathcal{D}_\mathrm{M}}(W)$ is where $\mathcal{D}_\mathrm{M} = \{d \in \mathcal{D} \mid \mathrm{just}(r) = \{\}\}$.

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Alternative Definitions

- Let $(\mathcal{D}, W)$ be a default theory, S a set of formulas. Let $\Gamma(S)$ be the smallest set such that:
  1. $W \subseteq \Gamma(S)$,
  2. $\mathrm{Cn}(\Gamma(S)) = \Gamma(S)$
  3. if $\frac{\varphi : \psi_1, \ldots, \psi_n}{\gamma} \in \mathcal{D}$, $\varphi \in \Gamma(S)$, $\neg\psi_i \notin S$ then $\gamma \in \Gamma(S)$.

  E is an extension of $(\mathcal{D}, W)$ iff $E = \Gamma(E)$
- Define, for $(\mathcal{D}, W)$, a sequence
  $E_0 = W$, and for $i \geq 0$
  $E_{i+1} = \mathrm{Cn}(E_i) \cup \{\gamma \mid \frac{\varphi : \psi_1, \ldots, \psi_n}{\gamma} \in \mathcal{D}, \varphi \in E_i, \neg\psi_i \notin E\}$.
  E is an extension of $(\mathcal{D}, W)$ iff $E = \bigcup_{i=0}^{\infty} E_i$.

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Default reasoning under alternative belief sets:

- ➤ A cautious approach: φ is a *sceptical* consequence of $(\mathcal{D}, W)$ iff φ is in all extensions of $(\mathcal{D}, W)$
- ➤ A brave approach: φ is a *credulous* consequence of $(\mathcal{D}, W)$ iff φ is in at least one extension of $(\mathcal{D}, W)$.

Example. $\mathcal{D}$: $\dfrac{q(n) : p(n)}{p(n)}$ $\quad$ W: $q(n)$

$\dfrac{r(n) : \neg p(n)}{\neg p(n)}$ $\quad$ $r(n)$

- $p(n)$ is credulous consequences of $(\mathcal{D}, W)$, but not a sceptical one because there is an extension contain $p(n)$ as well as one not containing it.
- Similarly for $\neg p(n)$.
- $r(n)$ is a sceptical consequence of $(\mathcal{D}, W)$.

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First-order default reasoning

- ➤ Various alternatives to interpret open defaults (defaults with free variables): $\dfrac{quaker(X) : pacifist(X)}{pacifist(X)}$
- ➤ We use a simplified setting where an open default is a schema (shorthand) for it closed instances.
- ➤ See, e.g., [Reiter, 1980], [Kaminski, 1995] for more complicated methods and detailed investigations.

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Finitary characterization of extensions

- ➤ Default logic seems to be very hard to handle computationally as

  (i) the key objects (extensions) are infinite set of formulae

  (ii) the definition is non-constructive (fixed point equation).
- ➤ A finitary characterization:

  (i) Represents an extension finitely.

  (ii) Basis for automated reasoning methods (checking whether a representation corresponds to an extension).

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Generating defaults

- ➤ Each extension of a default theory $(\mathcal{D}, W)$ has a set of generating defaults $\mathrm{GD} \subseteq \mathcal{D}$, i.e., if E is an extension of $(\mathcal{D}, W)$, then $E = \mathrm{Cn}(W \cup \{\mathrm{cons}(d) \mid d \in \mathrm{GD}\})$ where $\mathrm{GD} = \{d \in \mathcal{D} \mid \mathrm{pre}(d) \in E$ and for all $\psi \in \mathrm{just}(d), \neg\psi \notin E\}$.
- ➤ Hence, each extension corresponds to a subset of defaults but which subsets correspond to extensions?

  ☞ Grounded subsets where exactly all applicable default are included.
- ➤ A set of defaults $\mathcal{D}$ is grounded in a set formulae W iff for all $d \in \mathcal{D}$, $\mathrm{pre}(d) \in \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D})}(W)$ where $\mathrm{Mon}(\mathcal{D}) = \{\frac{\mathrm{pre}(d)}{\mathrm{cons}(d)} \mid d \in \mathcal{D}\}$.

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Theorem. E is an extension of $(\mathcal{D}, W)$ iff
$E = \mathrm{Cn}(W \cup \{\mathrm{cons}(d) \mid d \in \mathcal{D}'\})$ for defaults $\mathcal{D}' \subseteq \mathcal{D}$ such that
1. $\mathcal{D}'$ is grounded in W and
2. for all $d \in \mathcal{D}$: $d \in \mathcal{D}'$ iff
(i) $\mathrm{pre}(d) \in E$ and (ii) for all $\psi \in \mathrm{just}(d), \neg\psi \notin E$.

Example. Consider $(\mathcal{D}, W)$ with
$\mathcal{D} = \{q(n) : p(n)/p(n), r(n) : \neg p(n)/\neg p(n)\}$, $W = \{q(n), r(n)\}$

$\{\frac{r(n) : \neg p(n)}{\neg p(n)}\}$ is a set of generating defaults for the extension
$E = \mathrm{Cn}(W \cup \{\neg p(n)\})$ because it is grounded and
$r(n) \in E, \neg\neg p(n) \notin E, \neg p(n) \in E$ implying that (2.) holds, too.

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Full sets

- ➤ Default rules: monotonic rules guarded with justifications.
  ☞ A set of justifications yields an extension.
- ➤ Monotonic rules enabled by a set of justifications:
  $\mathrm{Mon}(\mathcal{D}, F) = \{\frac{\mathrm{pre}(d)}{\mathrm{cons}(d)} \mid d \in \mathcal{D}, \mathrm{just}(d) \subseteq F\}$.

Definition. For a default theory $(\mathcal{D}, W)$, a set of justifications
$F \subseteq \mathrm{just}(\mathcal{D})$ is $(\mathcal{D}, W)$-full iff for all $\psi \in \mathrm{just}(\mathcal{D})$:
$\psi \in F$ iff $\neg\psi \notin \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$

Example. Consider $(\mathcal{D}, W)$ with
$\mathcal{D} = \{q(n) : p(n)/p(n), r(n) : \neg p(n)/\neg p(n)\}$, $W = \{q(n), r(n)\}$

$F_1 = \{\neg p(n)\}$ is $(\mathcal{D}, W)$-full as $\mathrm{Mon}(\mathcal{D}, F_1) = \{\frac{r(n)}{\neg p(n)}\}$ and
$\neg\neg p(n) \notin \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F_1)}(W)$ and $\neg p(n) \in \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F_1)}(W)$.

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Theorem. Let $(\mathcal{D}, W)$ a default theory.

1. If $F \subseteq \mathrm{just}(\mathcal{D})$ is $(\mathcal{D}, W)$-full, then $\mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$ is an extension of $(\mathcal{D}, W)$.
2. If E is an extension of $(\mathcal{D}, W)$, then
   $F = \{\psi \in \mathrm{just}(\mathcal{D}) \mid \neg\psi \notin E\}$ is $(\mathcal{D}, W)$-full and
   $E = \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$.

Example. Consider $(\mathcal{D}, W)$ with
$\mathcal{D} = \{q(n) : p(n)/p(n), r(n) : \neg p(n)/\neg p(n)\}$, $W = \{q(n), r(n)\}$

Now $\mathrm{just}(\mathcal{D}) = \{p(n), \neg p(n)\}$. It has 4 subsets which are candidates for extensions.

$F_1 = \{\neg p(n)\}$ is $(\mathcal{D}, W)$-full, $\mathrm{Mon}(\mathcal{D}, F_1) = \{\frac{r(n)}{\neg p(n)}\}$ and
$\mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F_1)}(W)$ is the corresponding extension.

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Full sets

- ➤ Extension checking: does a subset F of justifications correspond to an extension

  Construct $\mathrm{Mon}(\mathcal{D}, F)$
  For every $\psi \in \mathrm{just}(\mathcal{D})$ **do**
    If $\neg\psi \in \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$ and $\psi \in F$ or
      $\neg\psi \notin \mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$ and $\psi \notin F$ **then**
      output "F is not $(\mathcal{D}, W)$-full"; stop
    end if
  output "F is $(\mathcal{D}, W)$-full"

- ➤ A naive algorithm for finding all extensions $(\mathcal{D}, W)$:

  For every subset F of $\mathrm{just}(\mathcal{D})$ **do**
    If F is $(\mathcal{D}, W)$-full **then**
      output $\mathrm{Cn}_{\mathrm{Mon}(\mathcal{D},F)}(W)$