# A Linear Transformation from Prioritized Circumscription to Disjunctive Logic Programming*

Emilia Oikarinen** and Tomi Janhunen

Helsinki University of Technology, P.O. Box 5400, FI-02015 TKK, Finland

**Introduction.** The stable model semantics of *disjunctive logic programs* (DLPs) is based on *minimal models* which assign atoms false by default. While this feature is highly useful—leading to concise problem encodings—it occasionally renders knowledge representation with disjunctive rules difficult. Reiter-style *minimal diagnoses* [1] provide a good example in this respect. This problem can be alleviated by a more refined control of minimization provided by *parallel circumscription* [2] which allows certain atoms to *vary* or to have *fixed* truth values. The scheme of *prioritized circumscription* [3, 2] generalizes this setting with priority classes for atoms being minimized. Our aim is to bring these enhancements of minimality to the realm of disjunctive logic programming. We strive for a translation-based approach where varying and fixed atoms, as well as priority classes are effectively removed from representations by transformations. We have already addressed parallel circumscription and provided a *linear* and *faithful* but *non-modular* translation [4]. Here we present a similar transformation for prioritized circumscription, extend our implementation [5], and report preliminary experiments.

**Circumscription.** In the sequel, we consider the two forms of circumscription in the propositional case. Given a theory $\mathit{\Pi}$ represented as a *positive* DLP, the purpose of a prioritized circumscription $\mathrm{Circ}(\mathit{\Pi}, P_1 > \ldots > P_k, V, F)$ [1] of $\mathit{\Pi}$ is to falsify atoms in each set $P_i$, with a decreasing level of priority $0 < i \leq k$, as far as possible. Meanwhile the truth values of atoms in $V$ may *vary* freely and the truth values of atoms in $F$ are kept *fixed*. These objectives can be captured using a notion of minimality as follows.

**Definition 1.** *A model $M \models \mathit{\Pi}$ of a positive DLP $\mathit{\Pi}$ is $\langle P_1 > \ldots > P_k, V, F\rangle$-minimal iff there is no $N \models \mathit{\Pi}$ such that (i) $N \cap P_1 \subset M \cap P_1$, or $N \cap (P_1 \cup \ldots \cup P_{i-1}) = M \cap (P_1 \cup \ldots \cup P_{i-1})$ and $N \cap P_i \subset M \cap P_i$ for some $1 < i \leq k$; and (ii) $N \cap F = M \cap F$.*

The parallel circumscription $\mathrm{Circ}(\mathit{\Pi}, P, V, F)$ of $\mathit{\Pi}$ is obtained as a special case of Definition 1 ($k = 1$). In addition, the $\langle P_1 > \ldots > P_k, V, F\rangle$-minimality of $M \models \mathit{\Pi}$ is captured by the unsatisfiability of a translation $\mathrm{Tr}_{\mathrm{UNSAT}}(\mathit{\Pi}, P_1 > \ldots > P_k, F, M) =$

$$\begin{aligned}\{(A \setminus F) \leftarrow (B \setminus F) \mid A \leftarrow B \in \mathit{\Pi},\, M \not\models \textstyle\bigvee(A \cap F), \text{ and } M \models B \cap F\} \cup \\ \{e_0 \leftarrow\} \cup \textstyle\bigcup_{i=1}^{k}\{e_i \leftarrow (P_i \cap M) \cup \{e_{i-1}\}\} \cup \{\bot \leftarrow e_k\} \cup \\ \textstyle\bigcup_{i=1}^{k}\{\bot \leftarrow a, e_{i-1} \mid a \in P_i \setminus M\}.^2 \quad (1)\end{aligned}$$

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[1] The sets of atoms $P_1, \ldots, P_k$, $V$, and $F$ are mutually disjoint and cover all atoms of $\mathit{\Pi}$.

[2] Here $e_0$ and $e_1, \ldots, e_k$, which correspond to priority classes $P_1, \ldots, P_k$, are new atoms.

**Translation-Based Approach**. In [4], we present a transformation that captures the models of a parallel circumscription $\mathrm{Circ}(\Pi, P, V, F)$ with the stable models of its translation. Prioritized circumscription is handled by translating it to parallel circumscription using Lifschitz' (quadratic) scheme [5]. Here we extend the method from [4] for prioritized circumscription. The translation $\mathrm{Tr}_{\mathrm{circ2dlp}}(\Pi, P_1 > \ldots > P_k, V, F)$ consists of two DLP *modules* (cf. DLP-*functions* in [6]). The module $\mathrm{Tr}_{\mathrm{gen}}(\Pi)$ generates a model candidate for $\mathrm{Circ}(\Pi, P_1 > \ldots > P_k, V, F)$ roughly in the same way as in [4]. The module $\mathrm{Tr}_{\mathrm{min}}(\Pi)$ encodes the test for $\langle P_1 > \cdots > P_k, V, F\rangle$-minimality as an unsatisfiability check based on (1). When the two modules are joined as a single DLP, model candidates created by $\mathrm{Tr}_{\mathrm{gen}}(\Pi)$ are passed as input to $\mathrm{Tr}_{\mathrm{min}}(\Pi)$ for testing $\langle P_1 > \ldots > P_k, V, F\rangle$-minimality. As a consequence, the $\langle P_1 > \cdots > P_k, V, F\rangle$-minimal models $M$ of a positive DLP $\Pi$ and the stable models $N$ of $\mathrm{Tr}_{\mathrm{circ2dlp}}(\Pi, P_1 > \ldots > P_k, V, F)$ end up in a bijective correspondence such that $M = N \cap \mathrm{At}(\Pi)$.

**Experiments**. Our translator CIRC2DLP (v2.1)[3] implements $\mathrm{Tr}_{\mathrm{circ2dlp}}(\cdot)$. We use the problem of finding Reiter-style minimal diagnoses for digital circuits as the benchmark. For $k > 1$ priority classes for minimization, the performance of CIRC2DLP significantly improves the quadratic translation from [5]. On smaller instances the running times of CIRC2DLP (using DLV as back-end) and CIRCUM2 are very similar, but the memory consumption of CIRCUM2 becomes soon a bottleneck (over 512MB) as instances grow.

**Discussion**. The translation $\mathrm{Tr}_{\mathrm{circ2dlp}}(\cdot)$ improves its predecessors [4, 5] and it has a distinctive set of properties: (i) arbitrary propositional theories $\Pi$ subject to prioritized circumscription are covered, (ii) the translation $\mathrm{Tr}_{\mathrm{circ2dlp}}(\Pi, P_1 > \ldots > P_k, V, F)$ can be produced in linear time and space before computing any models, (iii) the minimal models of $\mathrm{Circ}(\Pi, P_1 > \ldots > P_k, V, F)$ and the stable models of its translation are in a bijective relationship, (iv) the signature $\mathrm{At}(\Pi)$ is preserved, and (v) there is no need for incremental updating. All previous transformations lack some of the features (i)–(v). In particular, those involving *characteristic clauses* and *loop formulas*, and the one underlying CIRCUM2, are worst-case exponential. Our first experiments indicate that CIRC2DLP combined with a disjunctive solver compares favorably with CIRCUM2.

## References


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[3] See `http://www.tcs.hut.fi/Software/circ2dlp/` for binaries and benchmarks.