We are interested
in programming tools
used for string generation
– such as macro processors
and parametric L systems
HOWEVER: even well-known
devices from formal
language theory can serve
as tutorial examples of
our framework

TREES

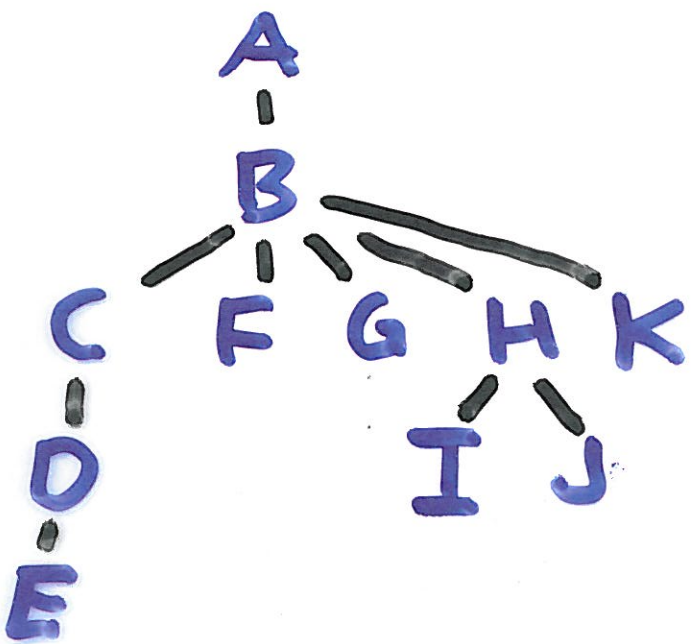
- finite, rooted, and ordered
- each node holds a letter

LETTERS

- nonterminals and terminals
- a word is a finite letter sequence

BELTS = TREE CROSS SECTIONS

Each leaf has exactly one ancestor included!



{A}

{B}

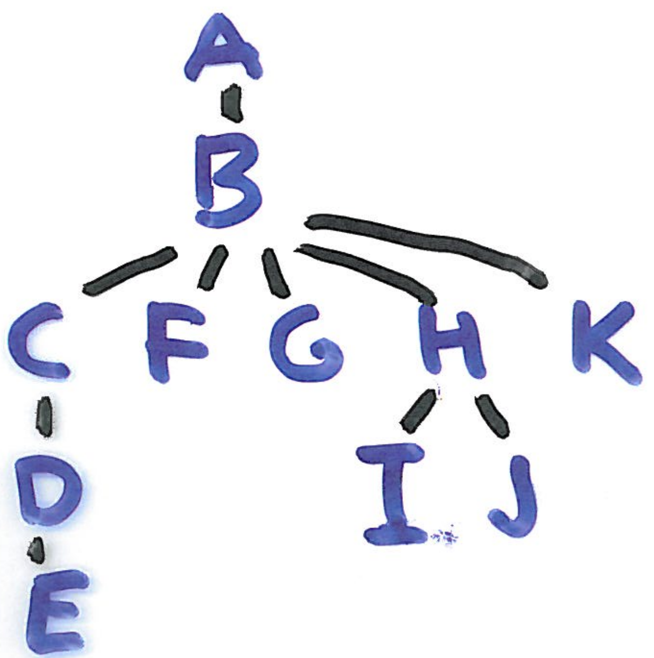
{C, F, G, H, K}

{D, F, G, H, K}

{E, F, G, I, J, K}

etc...

SPAN BETWEEN TWO TREE NODES



$\measuredangle(E,H) = \langle 3,1,1 \rangle$

$\measuredangle(H,E) = \langle 1,-1,3 \rangle$

$\measuredangle(E,G) = \langle 3,1,1 \rangle$

$\measuredangle(G,E) = \langle 1,-1,3 \rangle$

$\measuredangle(I,B) = \langle 2,0,0 \rangle$

$\measuredangle(B,I) = \langle 0,0,2 \rangle$

COMBS

A **comb** is any function

$$f : \{\ldots, -2, -1, 0, 1, 2, \ldots\} \to \{0, 1, 2, \ldots\} \cup \{\infty\}$$

with the restriction that

$$i \neq 0 \Rightarrow f(i) \neq 0$$

- There is a bijection between combs and belt-selectors

BELT-SELECTOR OPERATION

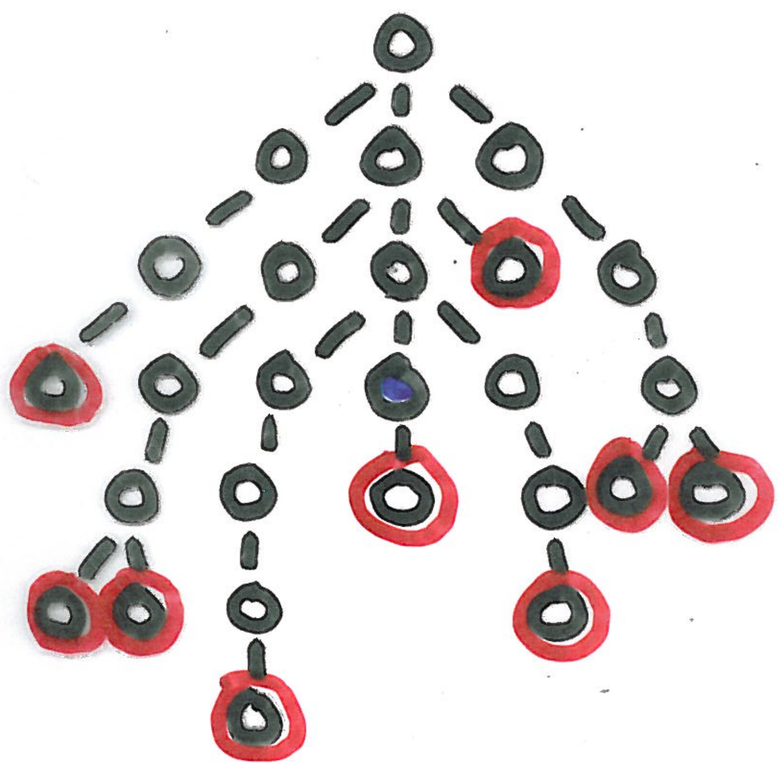
Let f be a comb, X a tree, $n \in X$

1. $n' \in X$ is chosen if it is not a proper ancestor of n and if $\measuredangle(n, n') = \langle i, d, f(i \times d) \rangle$ for some i and d
2. Every leaf with no ancestor already chosen is chosen

The belt-selector is settled at n if every leaf due to 2 holds a terminal

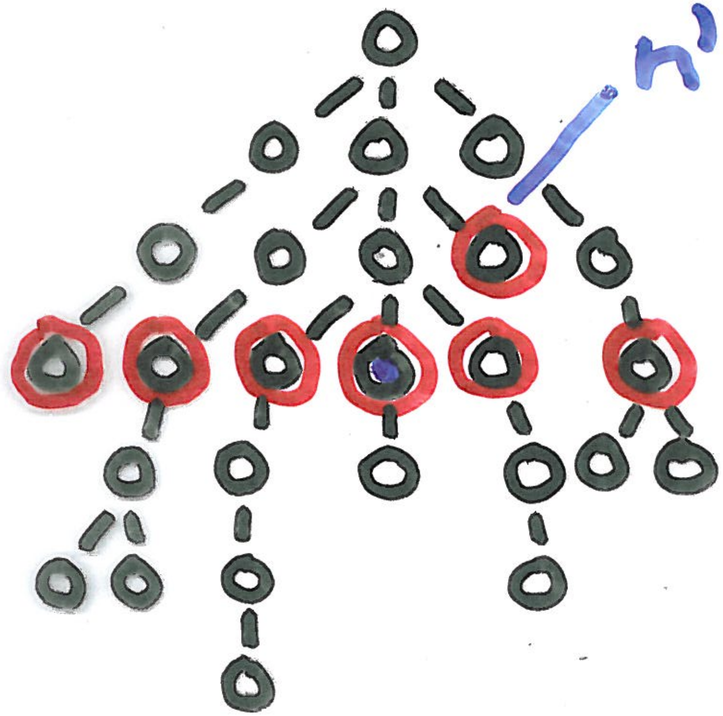
G_E

$f_{G_E}(i) = \infty$



O_c

$f_{O_c}(i) = |i|$

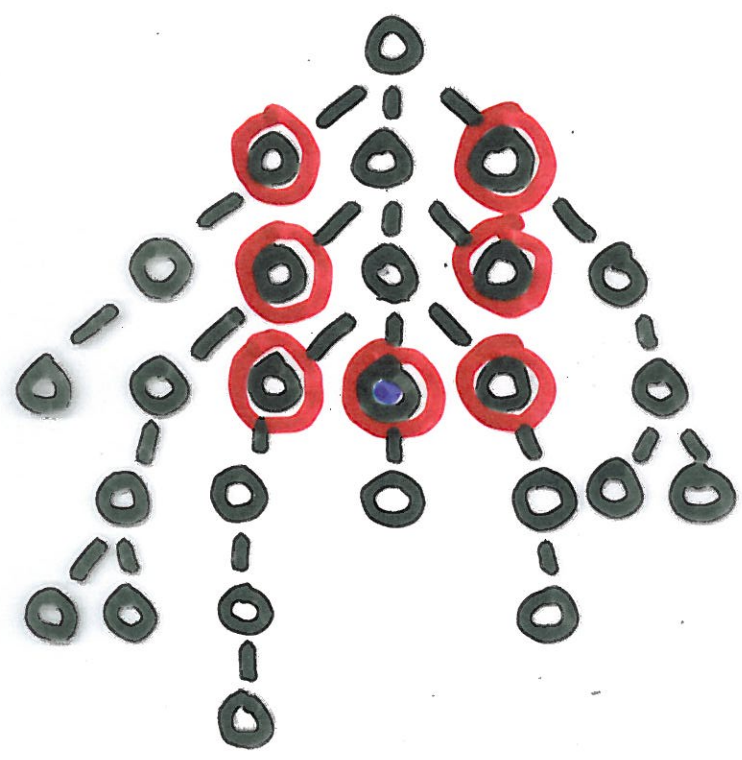


← not (quite) settled!

— unless n' holds a terminal!

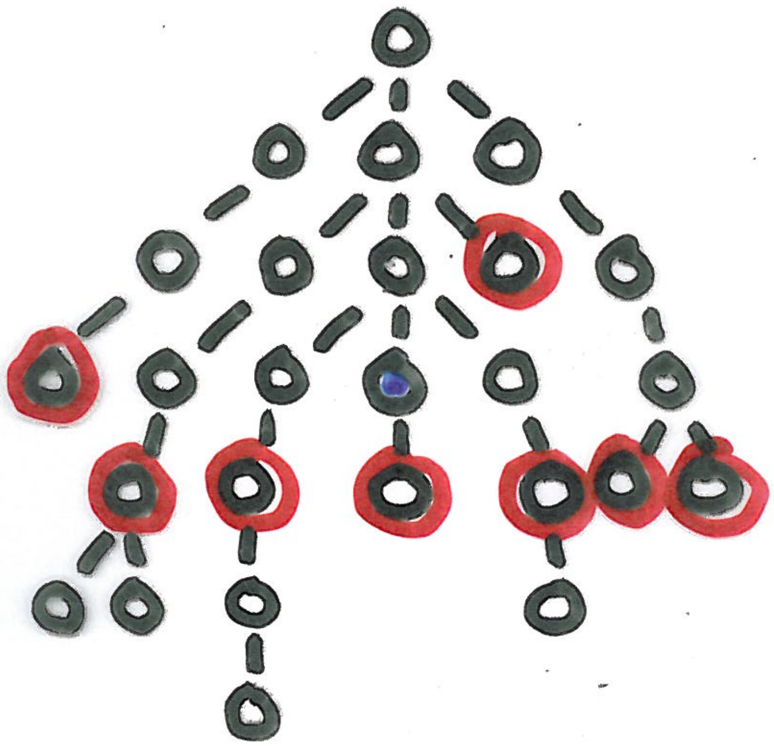
σ_I

$$f_{\sigma_I}(i) = \begin{cases} 0 & \text{when } i = 0 \\ 1 & \text{otherwise} \end{cases}$$

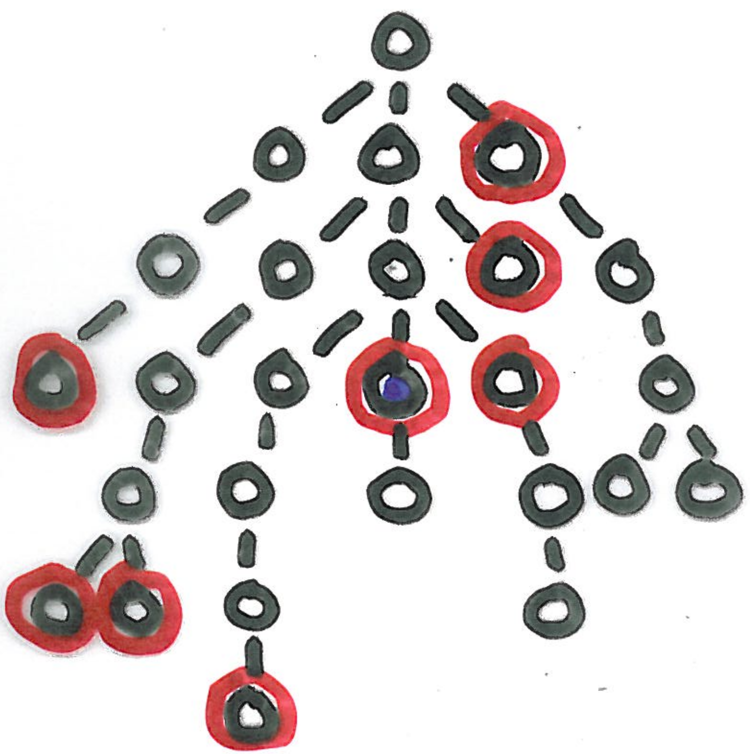


ΔG_c

$$f_{\Delta G_c}(i) = f_{G_c}(i) + 7$$



$$f_{G_E \| G_I}(i) = \begin{cases} f_{G_E}(i) & \text{when } i<0 \\ 0 & \text{when } i=0 \\ f_{G_I}(i) & \text{when } i>0 \end{cases}$$



$$G_E \| G_I$$

COMPONENTS OF A TETRASYSTEM

$$\langle V_N, V_T, c_s, r, \langle s_1, s_2, s_3, s_4 \rangle \rangle$$

V_N nonterminals

V_T terminals

c_s <u>seed-letter</u>

r <u>letter-refiner</u>

$\langle s_1, s_2, s_3, s_4 \rangle$ control <u>frame</u> (4 belt-selectors)

ON THE LETTER-REFINER

- may operate on an infinite alphabet
- may be unboundedly context-sensitive in both directions

- cannot ever replace a letter with an empty word
- may be <u>nondeterministic</u>

ON LETTER-REFINER NONDETERMINISM

Suppose $r(AB, C, DEF) = \{Q, PQR\}$

Then

$$A\ B\ C\ D\ E\ F$$

?

$$A\ B\ \boxed{Q}\ D\ E\ F \qquad A\ B\ \boxed{P\ Q\ R}\ D\ E\ F$$

ON TETRASYSTEM OPERATION

- rewriting is implemented as a tree generation process
- the initial tree consists of a single node holding the seed-letter

	when ready?	from where the data?
leaf expansion	fertileness S_1	context S_2
output extraction	S_3 matureness	S_4 output

	S_1	S_2	S_3	S_4
macro processors	$\sigma_E \| \sigma_I$	σ_E	σ_E	σ_E
Chomsky grammars	σ_I	σ_E	σ_E	σ_E
pure grammars	σ_I	σ_E	$\Delta\sigma_I$	σ_E
L systems	σ_C	σ_C	$\Delta\sigma_C$	$\Delta\sigma_C$

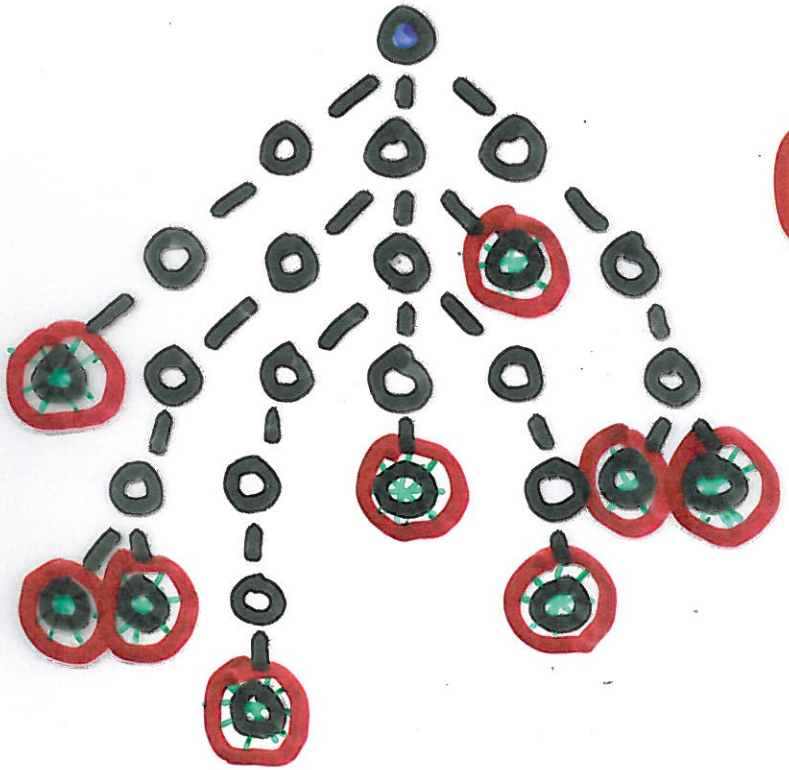
ON MACRO PROCESSING

- leaf expansion proceeds left-to-right and depth-first
- global variables are read from the left context
- when all leaves hold terminals, they constitute the output sequence

MACRO PROCESSING: LEAF EXPANSION

1. leaf expansion can take place when $s_1 = \sigma_E \| \sigma_I$ is settled at a nonterminal-lettered leaf
2. the refinement context is picked up by $s_2 = \sigma_E$

MACRO PROCESSING: OUTPUT EXTRACTION



3 output extraction can take place when $s_3 = \sigma_E$ is settled at a nonterminal-lettered node

4 the actual output sequence is picked up by $s_4 = \sigma_E$