

We are interested  
in programming tools  
used for string generation  
- such as macro processors  
and **Parametric** L systems

**HOWEVER:** even well-known  
devices from formal  
language theory can serve  
as tutorial examples of  
our framework

# TREES

- finite, rooted and ordered
- each node holds a letter

# LETTERS

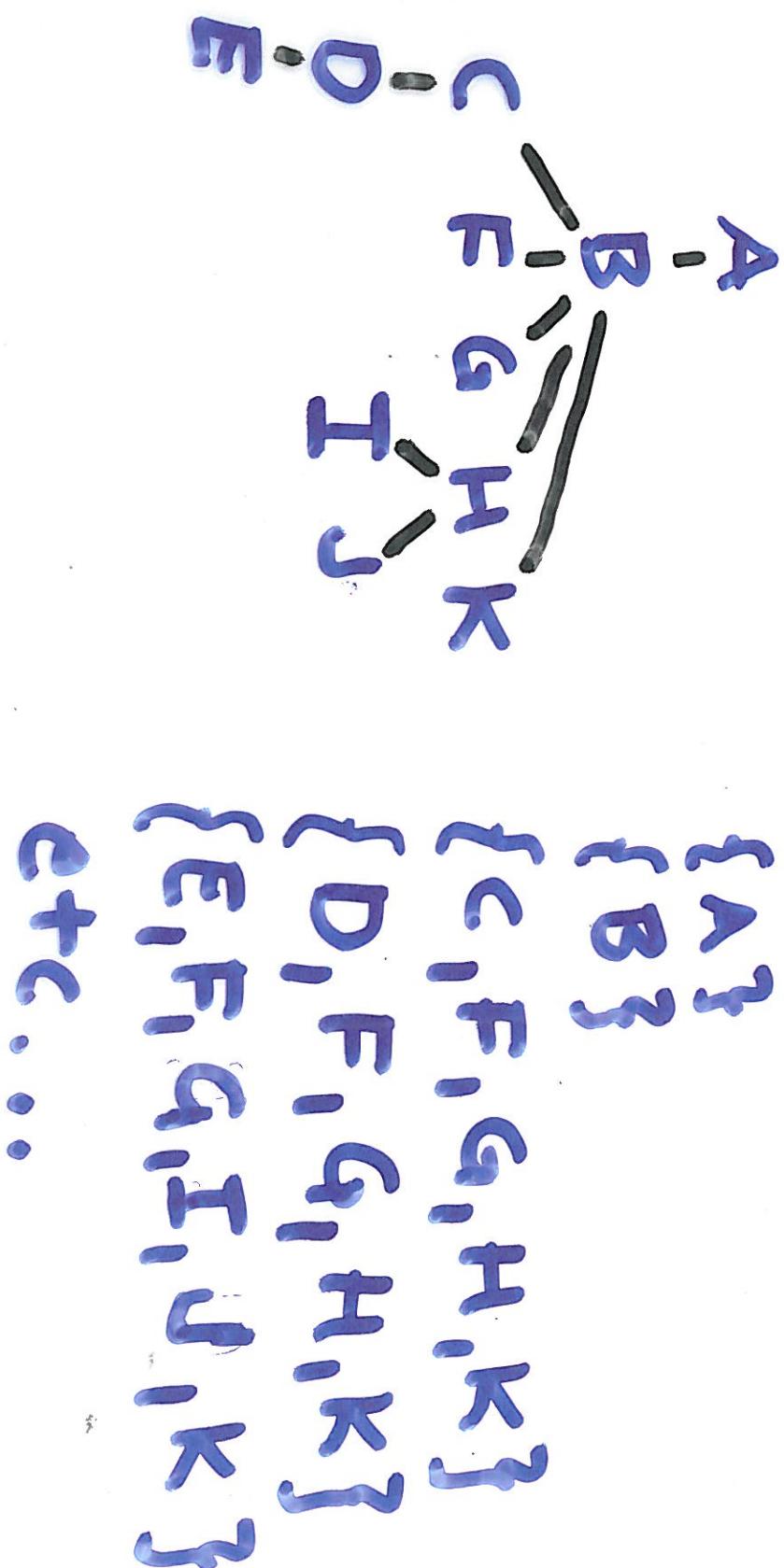
- nonterminals and terminals

- a word is a finite sequence

- letter sequences

# BELTS = TREE CROSS SECTIONS

Each leaf has exactly one ancestor included:



# SPAIN BETWEEN THE TRENDS

A hand-drawn diagram of a branched polymer chain. The backbone is represented by thick black lines. Side chains are shown as blue loops attached to the backbone at various points. The letters H, I, C, P, O, and N are written in blue along the backbone and side chains.

# COMBS

A comb is any function

$$f : \{ \dots, -2, -1, 0, 1, 2, \dots \}$$

$$\rightarrow \{ 0, 1, 2, \dots \} \cup \{ \infty \}$$

with the restriction that

$$i \neq 0 \Rightarrow f(i) \neq 0$$

- There is a bijection between combs and belt-selectors

# BELT-SELECTOR OPERATION

Let  $t$  be a comb-X-tree,  $\text{next}$

①  $\text{next}$  is chosen if it is not

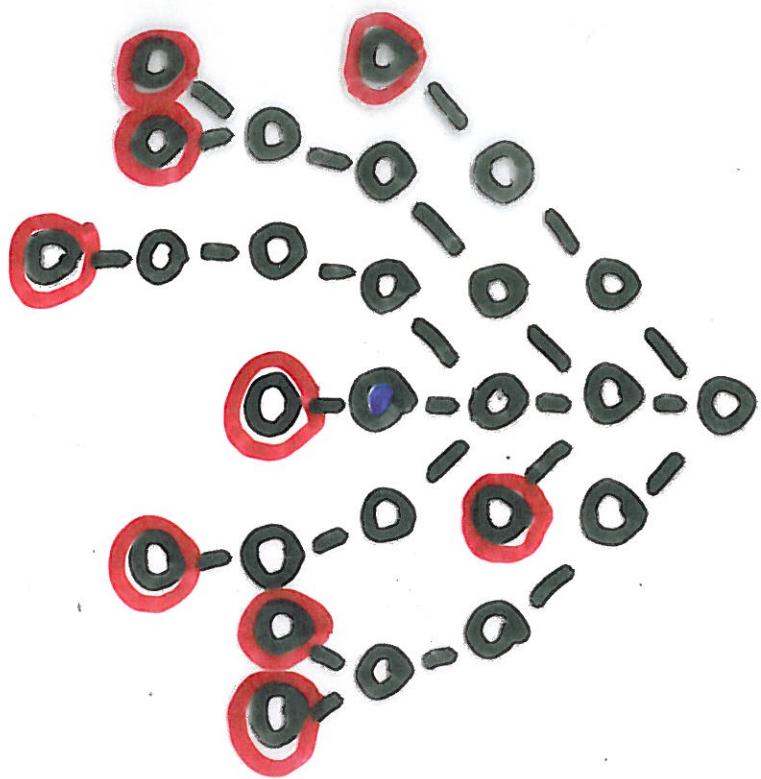
a proper ancestor of  $n$

and  $\delta(n, n') = \langle \cdot, \text{def}(i, x) \rangle$

for some  $i$  and  $x$

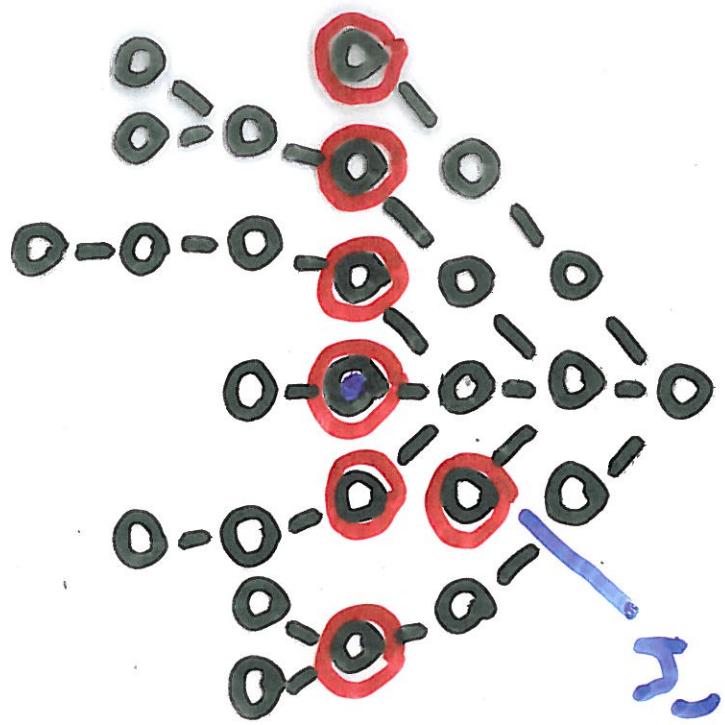
② Every leaf with no ancestor  
already chosen is chosen

The belt-selector is settled at  
if every leaf due to ②<sup>5</sup>  
holds a terminal



$G_E$

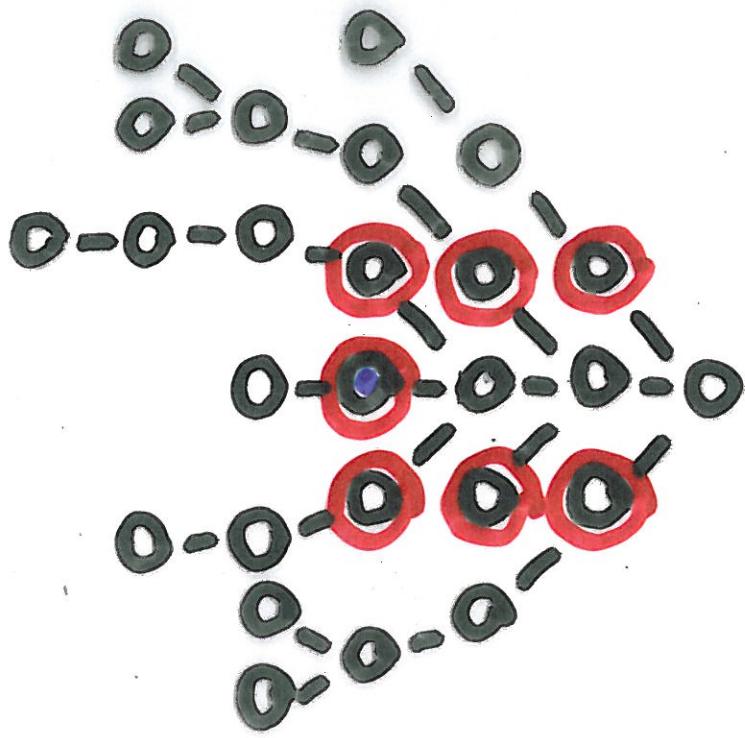
$$f_{G_E}(\cdot) = 8$$



$6c$

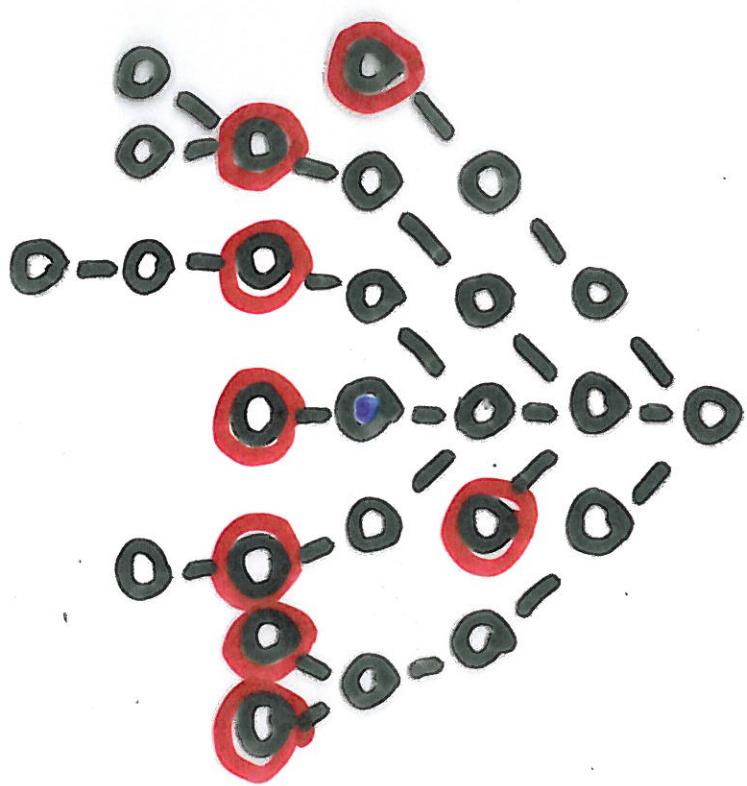
$$f_{6c}(i) = \underline{\underline{i}}$$

↑  
not  
(quite)  
settled  
— unless  
 $n_i$   
holds a terminal!



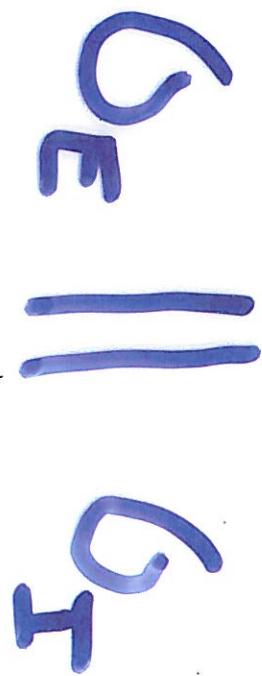
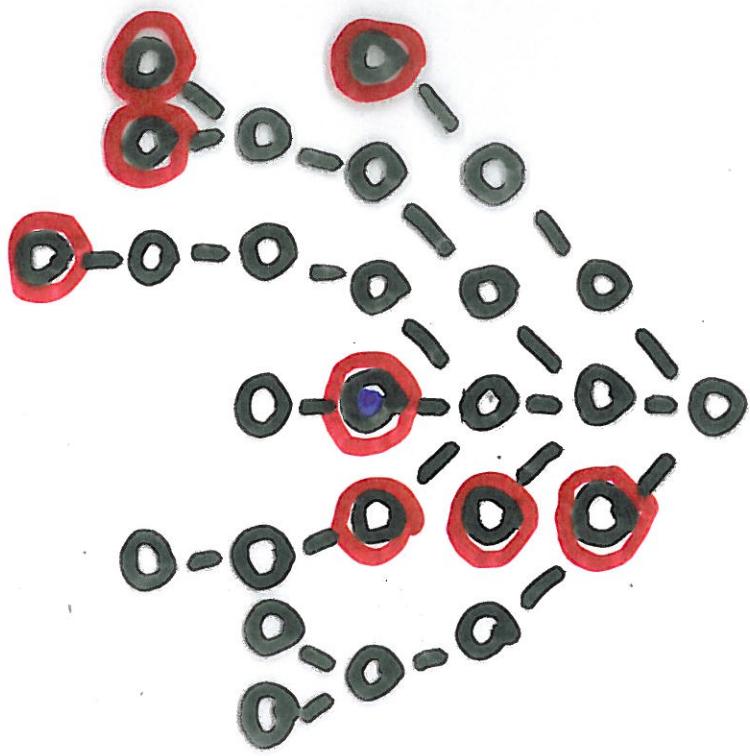
$\sigma_I$

$$f_{6I}(i) = \begin{cases} 0 & \text{when } i=0 \\ 1 & \text{otherwise} \end{cases}$$



$\Delta_{bc}$

$$f_{\Delta_{bc}}(z) = f_{bc}(z) +$$



$f_{GE} \parallel f_I(i) = \begin{cases} f_{GE}(i) & \text{when } i < 0 \\ f_{OI}(i) & \text{when } i = 0 \\ f_{GI}(i) & \text{when } i > 0 \end{cases}$

# COMPONENTS OF A TETRASYSTEM

$\langle V_N, V_T, CS, \Gamma \rangle \langle S_1, S_2, S_3, S_4 \rangle \langle \langle h_1, h_2, h_3, h_4 \rangle \rangle$

$V_N$   
nonterminals

$V_T$   
terminals

$CS$   
seed - letters

$\Gamma$   
letter - refiner

$\langle S_1, S_2, S_3, S_4 \rangle$

control frame  
(4 belt-selectors)

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## ON THE LETTER-REFINER

- may operate on an infinite alphabet
- may be unboundedly context-sensitive in both directions
- cannot ever replace a letter with an empty word
- may be nondeterministic

# ON LETTER-REFINER NONDETERMINISM

Suppose  $r(AB,C,DEF) = \{Q - PAR\}$

Then

A B C D E Π

?

A B Q D E Π

A B P Q R D E Π



# ON TETRAKSYSTEM OPERATION

- rewriting is implemented as a tree generation process
- the initial tree consists of a single node holding the seed-letter

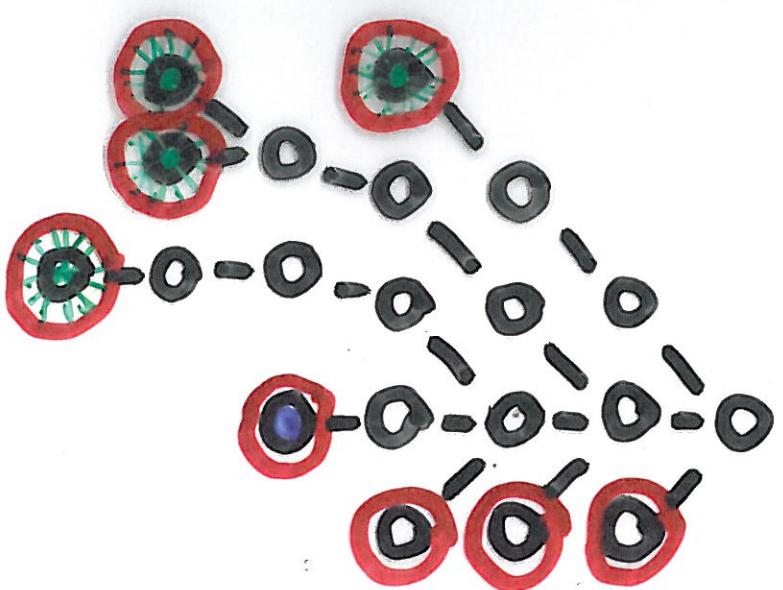
which  
reads?  
from where  
the data?

| Output         | maturity - ness | Output         | leaf expansion | fertile - ness | context        |
|----------------|-----------------|----------------|----------------|----------------|----------------|
| S <sub>3</sub> | S <sub>1</sub>  | S <sub>4</sub> | S <sub>2</sub> | S <sub>5</sub> | S <sub>6</sub> |
| S <sub>3</sub> | S <sub>1</sub>  | S <sub>4</sub> | S <sub>2</sub> | S <sub>5</sub> | S <sub>6</sub> |

|                  | $S_1$ | $S_2$ | $S_3$       | $S_4$       |
|------------------|-------|-------|-------------|-------------|
| macro processors | 6E    | 6E    | 6E          | 6E          |
| Chomsky Grammars | 6I    | 6I    | 6E          | 6E          |
| pure grammars    | 6E    | 6E    | 6E          | 6E          |
| L systems        | 6C    | 6C    | $\Delta 6C$ | $\Delta 6C$ |

## ON MACRO PROCESSING

- leaf expansion proceeds left-to-right and depth-first
- global variables are read from the left context
- when all leaves hold terminals, they constitute the output sequence



## MACRO PROCESSING: LEAF EXPANSION

①

Leaf expansion  
can take place  
when  $S_1 = 6E|6I$

is settled at a  
non-terminal - (letter)  
leaf

②

The refinement  
context is  
picked up by

$$S_2 = 6E$$

# **MICRO PROCESSING: OUTPUT EXTRACTION**

Output extraction  
can take place  
when  $S_3 = G_E$   
is selected at a  
non-terminal-  
lettered node  
the actual output  
sequence is  
picked up by

A hand-drawn diagram illustrating a crystal lattice structure. The structure is composed of a regular grid of red circles, each containing a smaller black circle. These red circles represent oxygen atoms, and the black circles represent metal ions. The connections between the lattice points are shown as black dashed lines, representing the covalent bonds in the crystal. Several points in the grid are highlighted with red circles, likely indicating specific sites or defects in the crystal structure.