DEFINING SOUNDNESS

① one has to choose an application-specific equivalence partition on words

– we say that w and w' have the same semantics if $w \overset{e}{\sim} w'$ for the equivalence e

② a letter-refiner is e-sound if it preserves the semantics, i.e.

$$w \in r(w_1, c, w_2) \Longrightarrow w_1 w w_2 \overset{e}{\sim} w_1 c w_2$$

EXAMPLE: Suppose our e has only two classes
- words in which B occurs at most once
- other words

$$\begin{tikzcd}[column sep=small] & AAAAA \arrow[dl] \arrow[dr] & \\ ABAAA & & AAABA \arrow[d] \\ & & ACCABA \end{tikzcd}$$

① AAAAA
ABAAA
AAABA
ACCABA

all with same semantics

Note: any c is transitive!

② ABABA however has different semantics

DEFINING SOUNDNESS cont'd

③ a tetrasystem is e-sound if every output word has the same semantics as the word constituted by the sole seed letter

④ Finally: a frame is sound if for every e and for every tetrasystem with the particular frame, the tetrasystem is e-sound if the letter-refiner is e-sound

SIMPLE SUFFICIENT CONDITIONS FOR FRAME SOUNDNESS

① $S_2 = S_4 = \sigma_I$
(curiosity)

② $S_2 = S_4 = \sigma_E$
(important!)

So the frames for macro processors, c-s Chomsky grammars, and pure grammars are all sound (as $S_2 = S_4 = \sigma_E$) — but the L system frame is actually not sound

ON NORMAL BELT-SELECTORS

- If we want soundness and thus have $s_2 = \sigma_E$, what choices do we have left?
- We are still free to choose a suitable root-to-leaves tree traversal order
- choosing a normal belt-selector for s_1 fully determines the order

MORE ON NORMALNESS

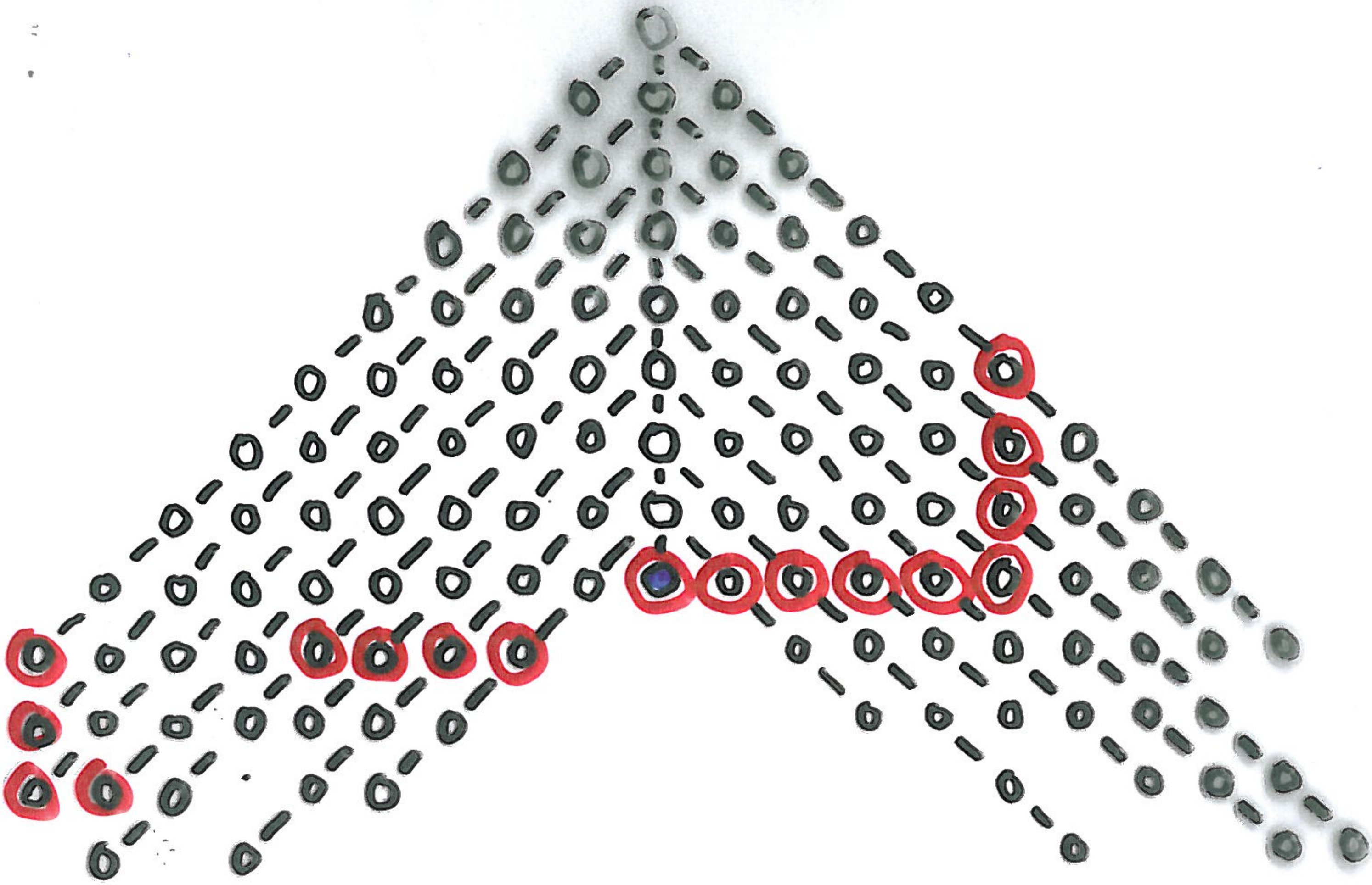
(we skip the exact comb-level characterization)

1. each normal belt-selector is weakly progressive (and some are strongly progressive)
2. but also: there is at most one fertile leaf (which prevents distributive progressiveness)
3. for the fertile leaf, the full leaf sequence is always selected
4. if s is normal, then $\forall i : \varphi_{s'}(i) \geq \varphi_s(i) \wedge \exists i : \varphi_{s'}(i) > \varphi_s(i)$ means s' is not weakly pr.

EXAMPLES

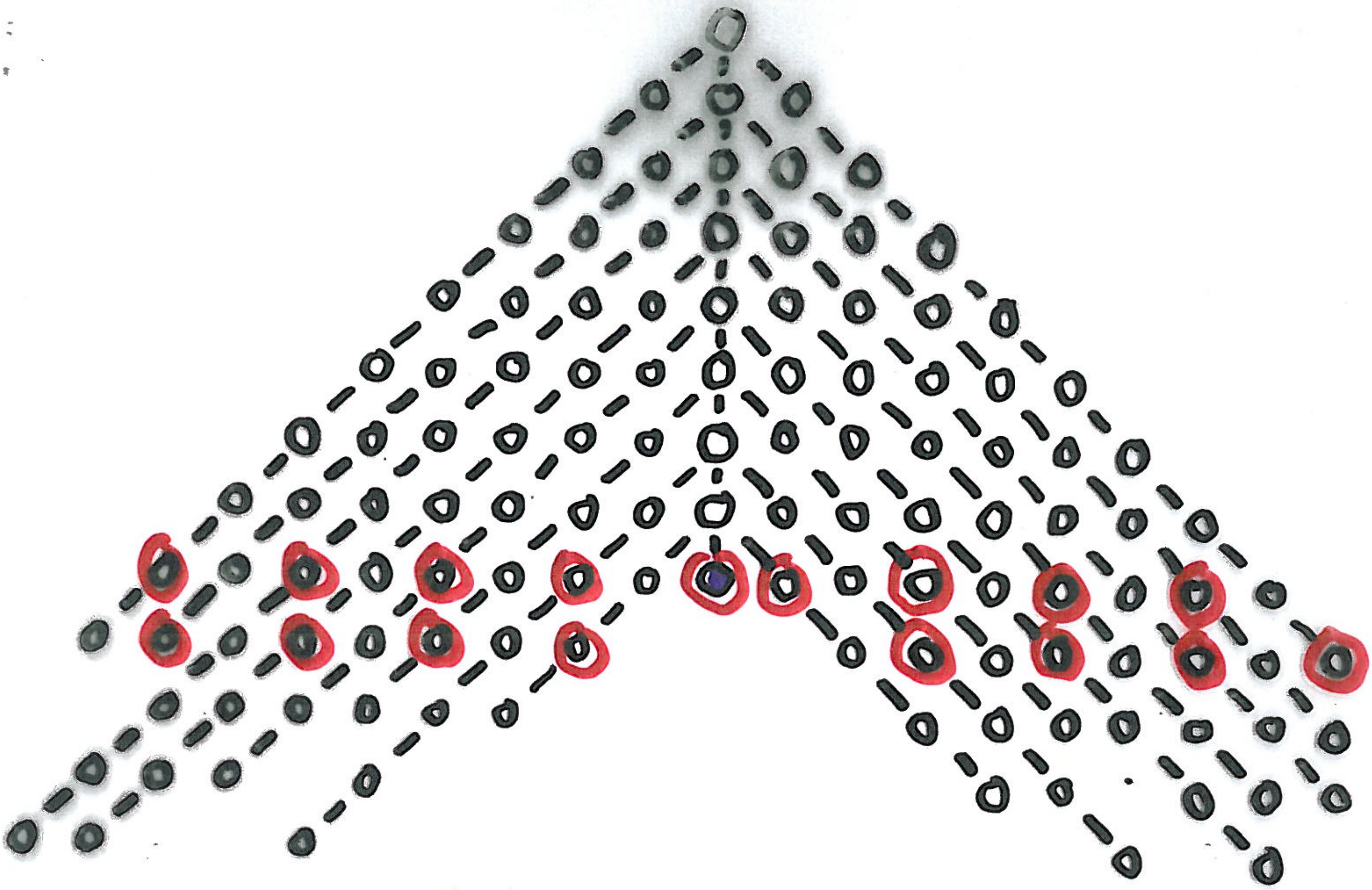
- $G_E \| G_I$ and $\Delta G_C \| G_C$ are normal (and so are their "mirror images")
- more examples follow...

- the set is actually uncountable (as is the set of functions from integers to any two distinct values)
- each belt-selector in the set specifies a unique tree traversal order

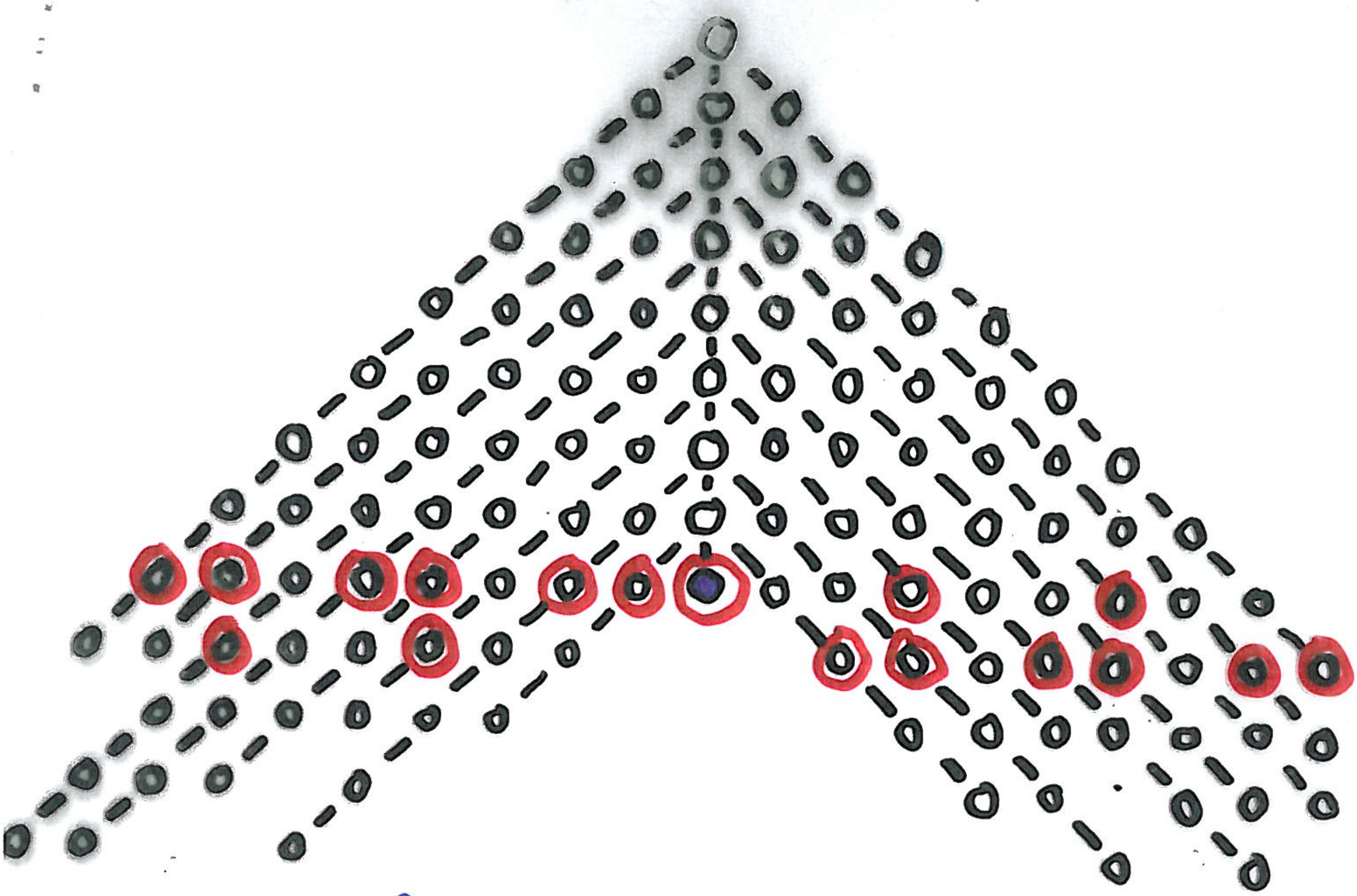


$$\varphi_5(i) = \begin{cases} \infty & \text{when } i \le -5 \\ -i+1 & \text{when } -5 < i < 0 \\ 0 & \text{when } i = 0 \\ i & \text{when } 0 < i < 5 \\ 5 & \text{when } i \ge 5 \end{cases}$$

Note: above, the argument node must hold a terminal if the tree has been produced with $s_1 = 5$



$$\varphi_s(i) = \begin{cases} 0 & \text{when } i = 0 \\ i & \text{when } i > 0 \text{ and } i \text{ odd} \\ i+1 & \text{when } i > 0 \text{ and } i \text{ even} \\ |i| + |i| + 1 - \varphi_s(-i) & \text{otherwise} \end{cases}$$



$$\varphi_s(i) = \begin{cases} 0 & \text{when } i = 0 \\ i & \text{when } i > 0 \\ & \text{and } i \bmod 3 = 0 \\ i+7 & \text{when } i > 0 \\ & \text{and } i \bmod 3 \neq 0 \\ |i| + |i| + 7 - \varphi_s(-i) & \\ & \text{otherwise} \end{cases}$$