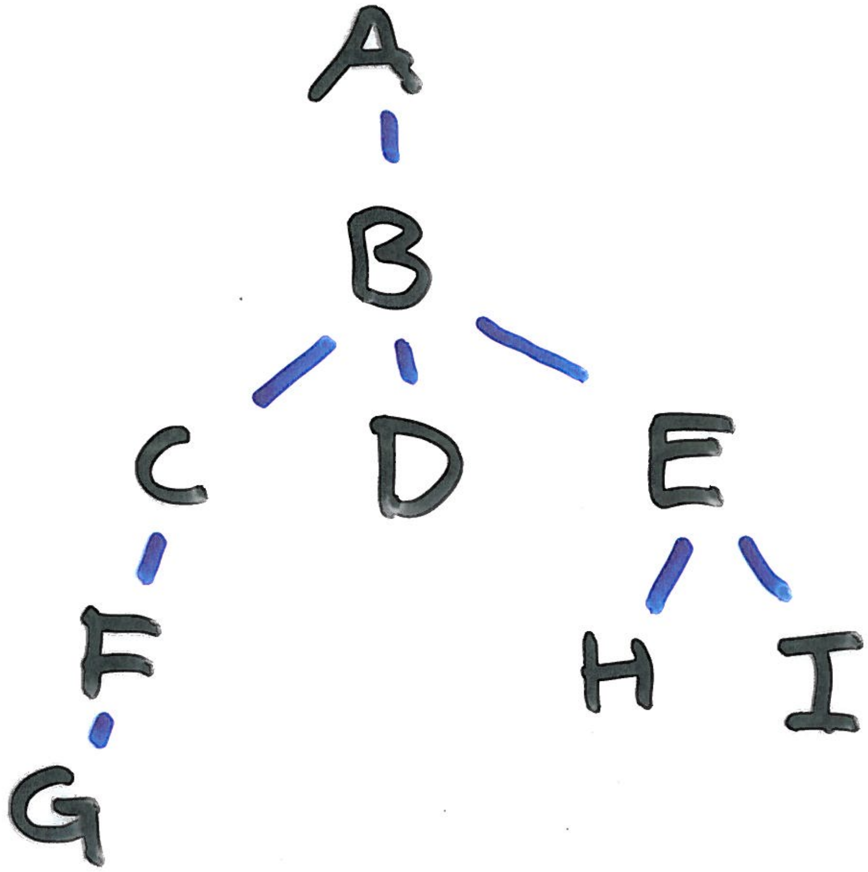
TODAY'S MENU

1. Introduction

2. Belt-selectors
    - belts
    - angles
    - (belt-assigners and) belt-selectors
    - bijection between belt-selectors and combs
    - examples

3. Tetrasystems

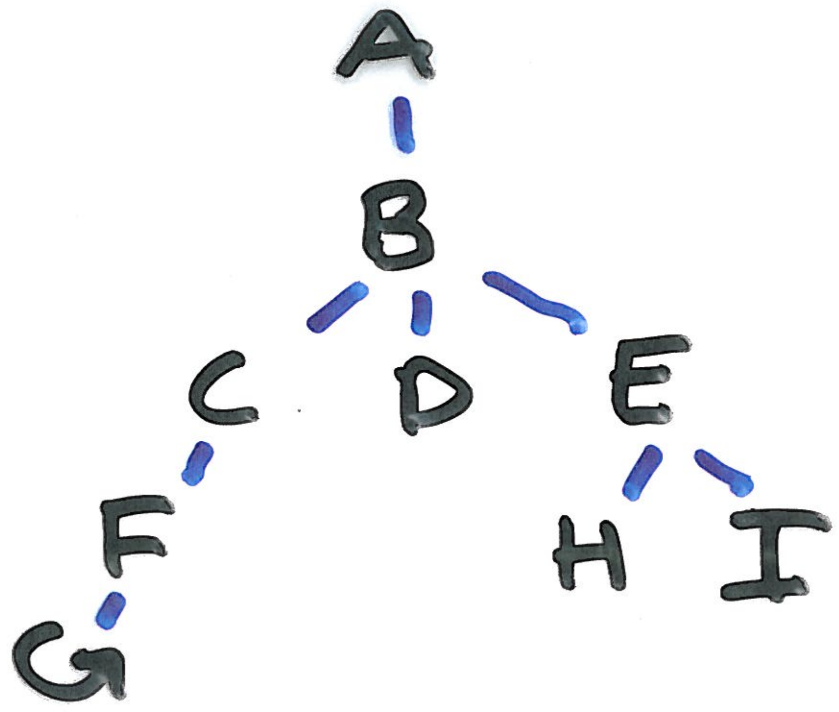
BELTS



$\{A\}, \{B\}, \{C, D, E\},$
$\{F, D, H, I\}, \{G, D, H, I\},$
etc.

Each leaf has exactly one ancestor in the belt!

ANGLES



$$\measuredangle(G,E) = \langle 3,1,1 \rangle$$
$$\measuredangle(E,G) = \langle 1,-1,3 \rangle$$

$$\measuredangle(G,D) = \langle 3,1,1 \rangle$$
$$\measuredangle(D,G) = \langle 1,-1,3 \rangle$$

$$\measuredangle(H,B) = \langle 2,0,0 \rangle$$
$$\measuredangle(B,H) = \langle 0,0,2 \rangle$$

An auxiliary definition:

a <u>belt-provider</u> is any function taking a tree and one of its nodes and returning a belt of the tree

- Like:

$s(X, n) = \{n_1, n_2, n_3\}$

MORE INTERESTING DEFINITIONS

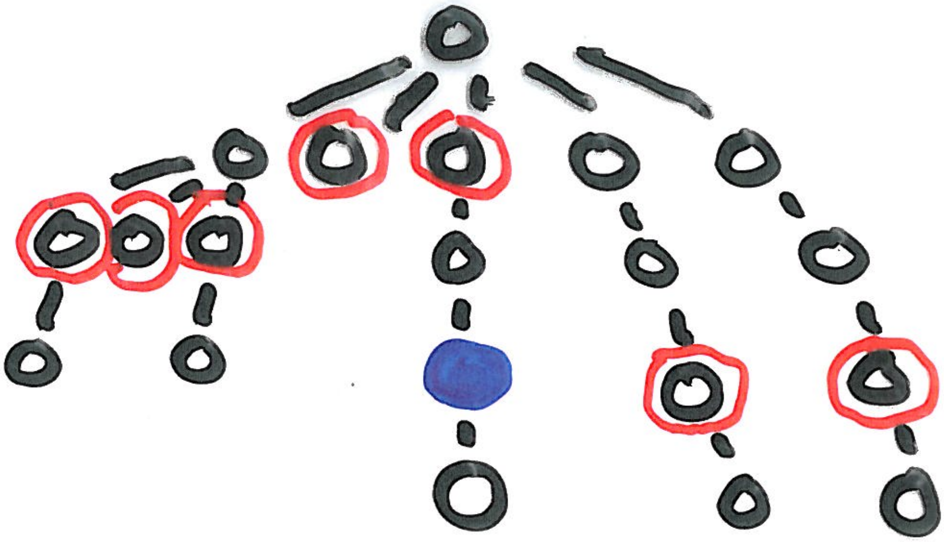
- a further condition on a belt-assigner:

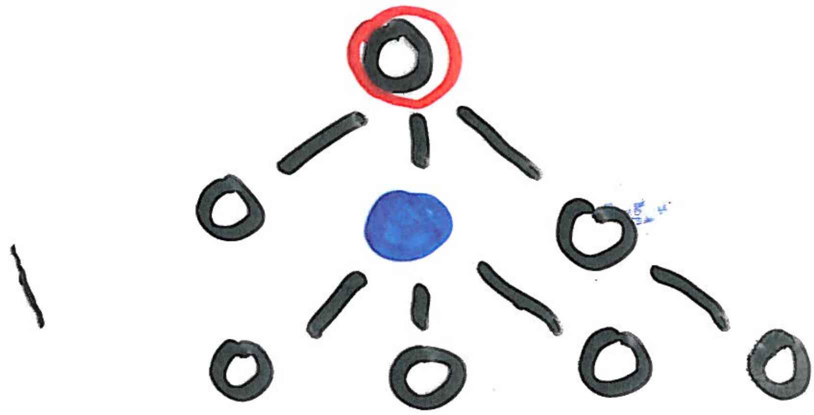
$n^* \in s(X,n) \Leftrightarrow m^* \in s(Y,m)$ if all the following conditions are met:

1. $\sphericalangle(n,n^*) = \sphericalangle(m,m^*)$
2. n^* is root $\Leftrightarrow$ m^* is root
3. n^* is leaf $\Leftrightarrow$ m^* is leaf

- a **belt-selector**: the above definition is tightened by dropping condition 2

A SAMPLE BELT-ASSIGNER

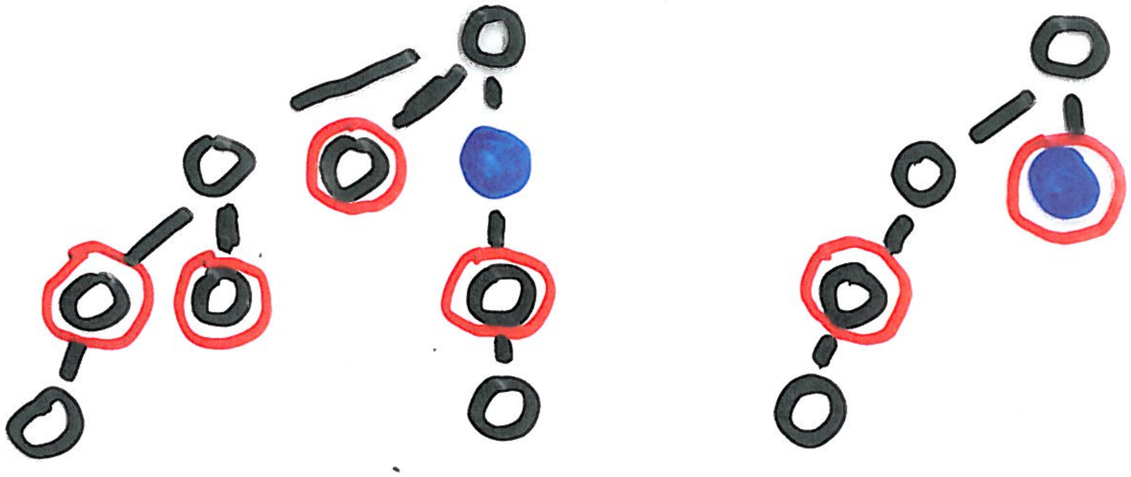




HINT:

$$f(-3) = 2$$
$$f(-2) = f(2) = 0$$
$$f(-1) = f(1) = -1$$
$$f(0) = -2$$
$$f(3) = 3$$

A SAMPLE BELT-SELECTOR



$$f(-1) = 2$$
$$f(0) = 1$$

<u>Note</u>: a belt-selector never selects a proper ancestor of the argument node

[Now we forget belt-providers and belt-assigners]

COMBS

- There is a bijection between belt-selectors and combs

A **comb** is any function
$$f: \{\ldots, -2, -1, 0, 1, 2, \ldots\} \to \{0, 1, 2, \ldots\} \cup \{\infty\}$$
with $f(i) = 0 \Rightarrow i = 0$

A given belt-selector s, whose comb is denoted as f, selects the following nodes for a given node n

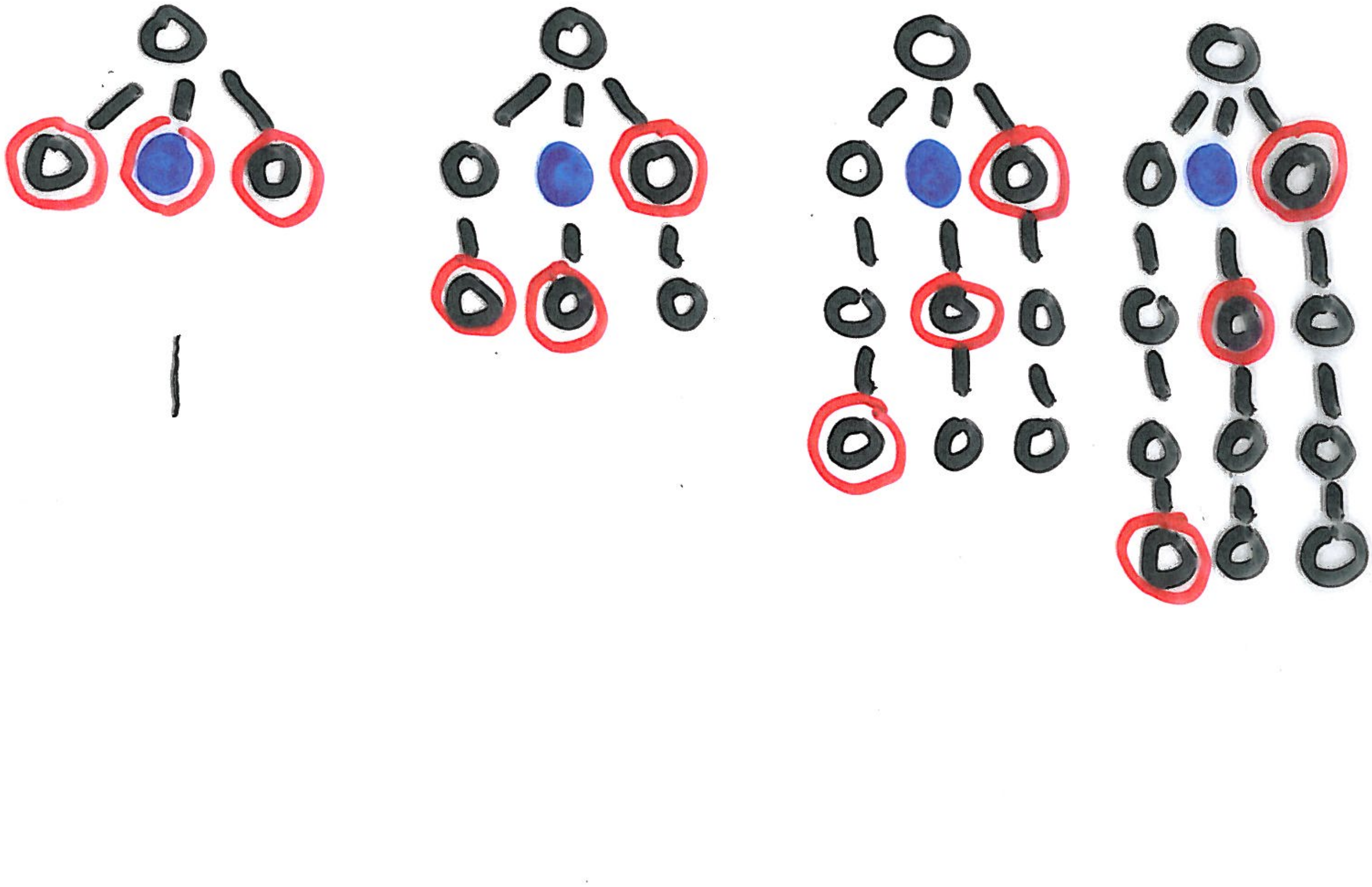
but not proper ancestors of n !!

1. all nodes n' with $\measuredangle(n, n') = \langle i, d, f(i*d) \rangle$
2. all leaves with no ancestor already selected by 1

EXAMPLE

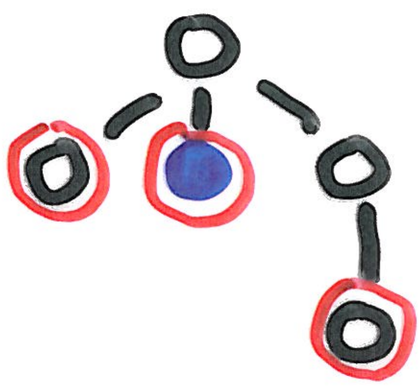
Suppose our belt-selector has the following Comb:

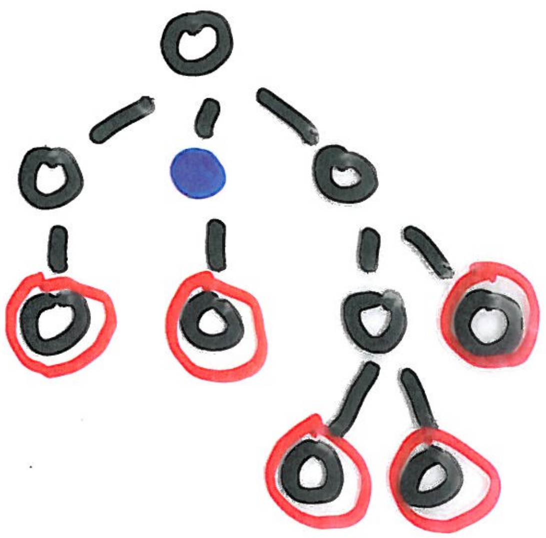
$$f(i) = \begin{cases} \infty & \text{when } i < 0 \\ 1 & \text{when } i \geq 0 \end{cases}$$



SPECIFIC BELT-SELECTORS
σ_E

$$\varphi_{\sigma_E}(i) = \infty$$



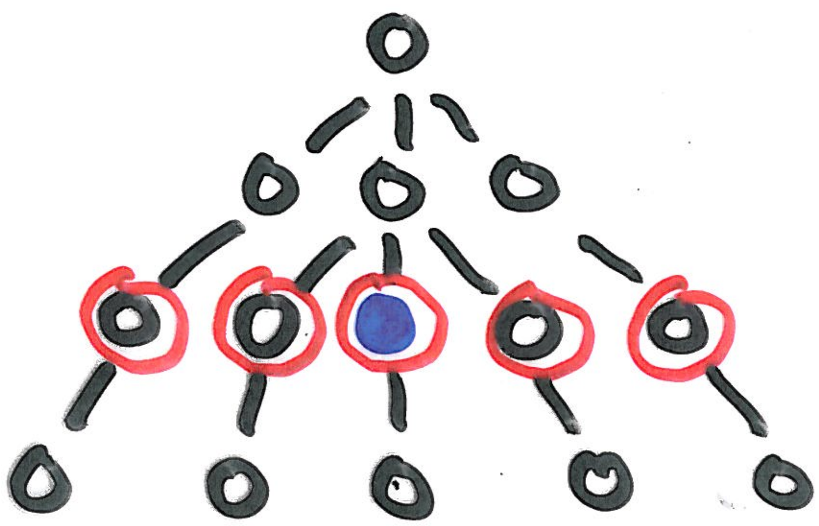


So σ_E always selects all the leaves!

ANOTHER SPECIFIC BELT-SELECTOR:

O_C

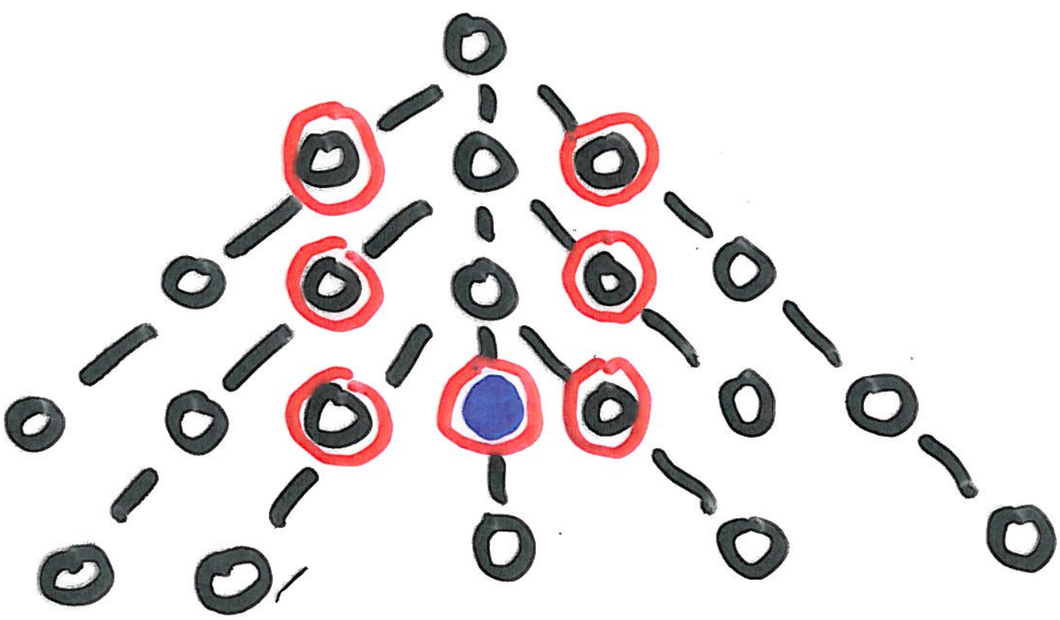
$$\varphi_{O_C}(i) = |i|$$



YET ANOTHER SPECIFIC BELT-SELECTOR:

$$\sigma_I$$

$$\varphi_{\sigma_I}(i) = \begin{cases} 0 & \text{when } i = 0 \\ 1 & \text{when } i \neq 0 \end{cases}$$

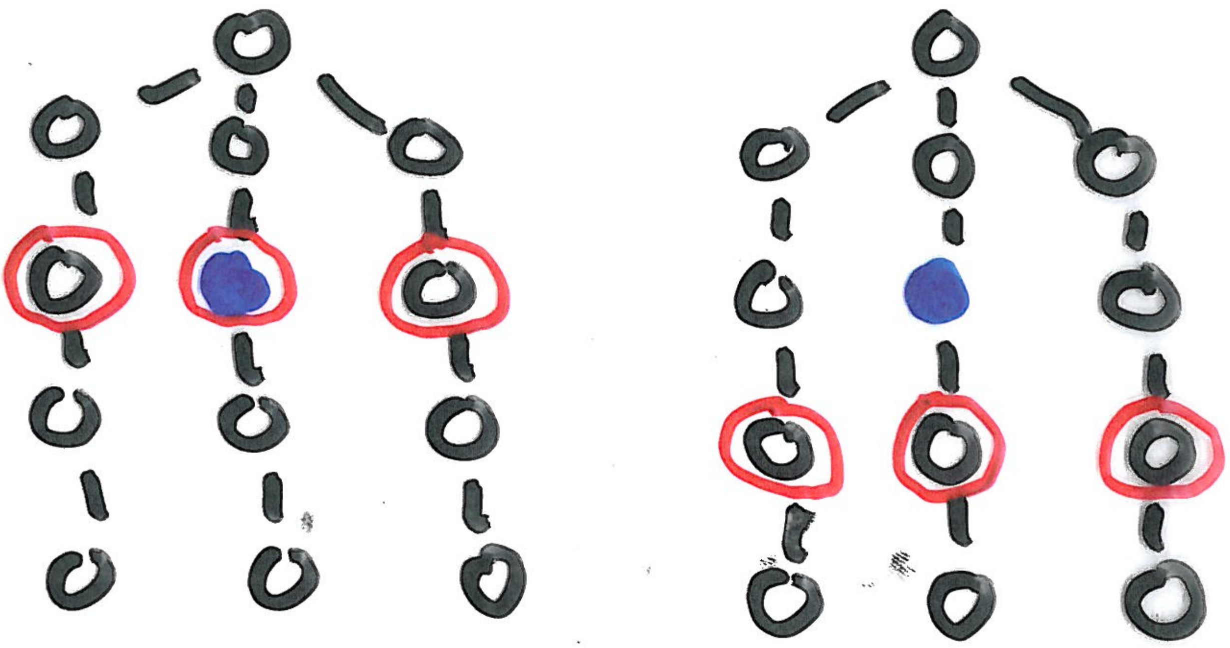


So σ_I is the least greedy possible (while σ_E is the most greedy)!

ENTER AN OPERATOR:

ΔS

$$\varphi_{\Delta\sigma_c}(i) = \varphi_{\sigma_c}(i) + 1$$
$$= |i| + 1$$

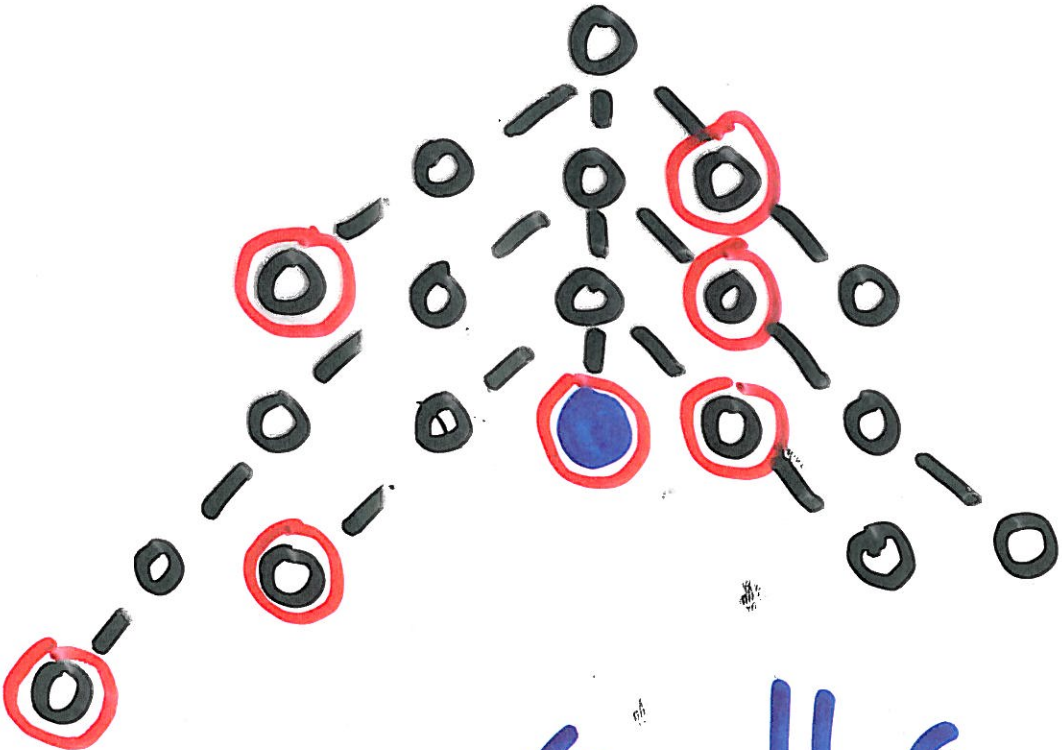


σ_c $\Delta\sigma_c$

FINALLY, A BINARY OPERATOR:

$$S_1 \parallel S_2$$

$$\varphi_{\sigma_E \parallel \sigma_I} = \begin{cases} \varphi_{\sigma_E}(i) & \text{when } i<0 \\ 0 & \text{when } i=0 \\ \varphi_{\sigma_I}(i) & \text{when } i>0 \end{cases}$$



$$\sigma_E \parallel \sigma_I$$